PROTON STRUCTURE
THE LAST LIGHT PARTON

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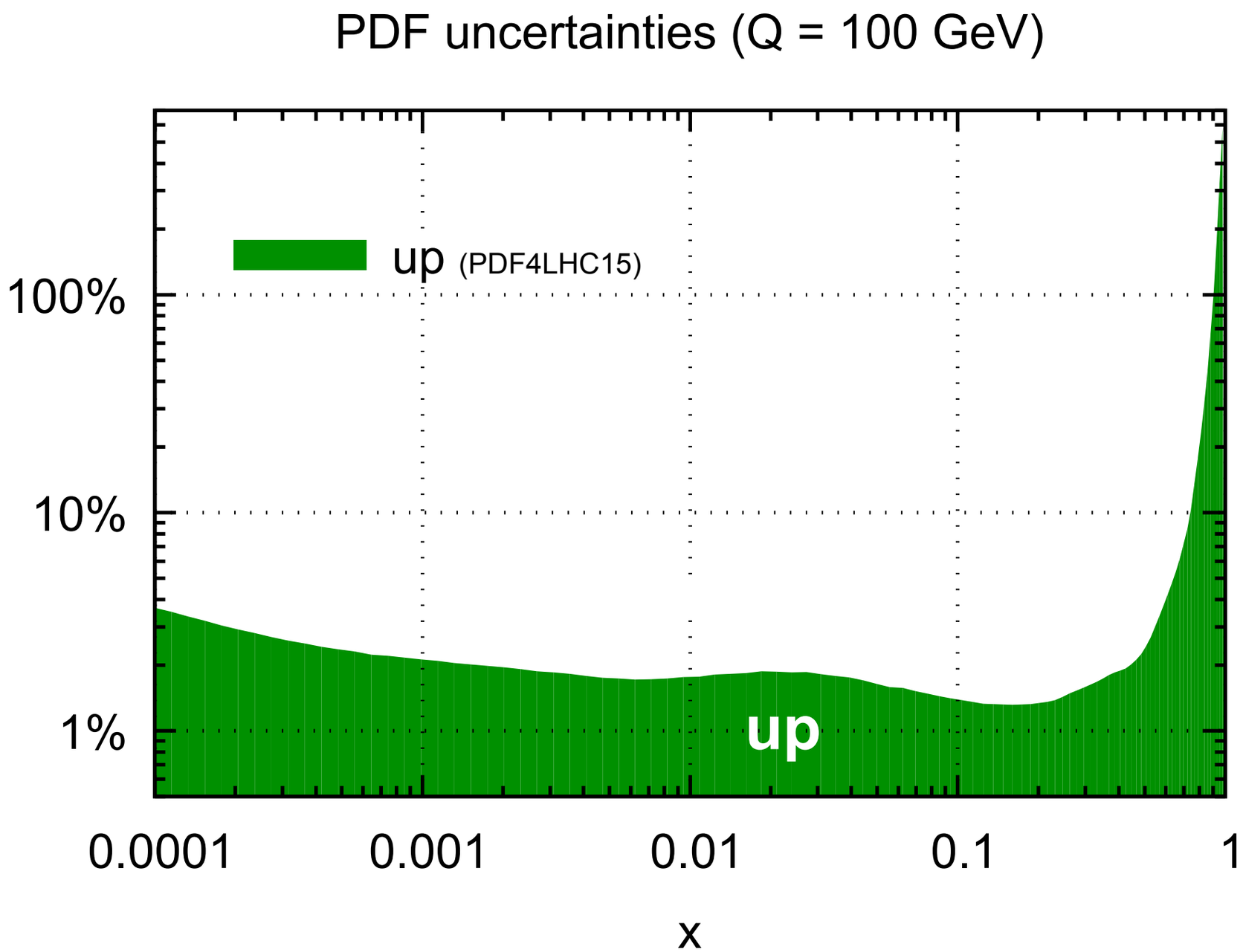
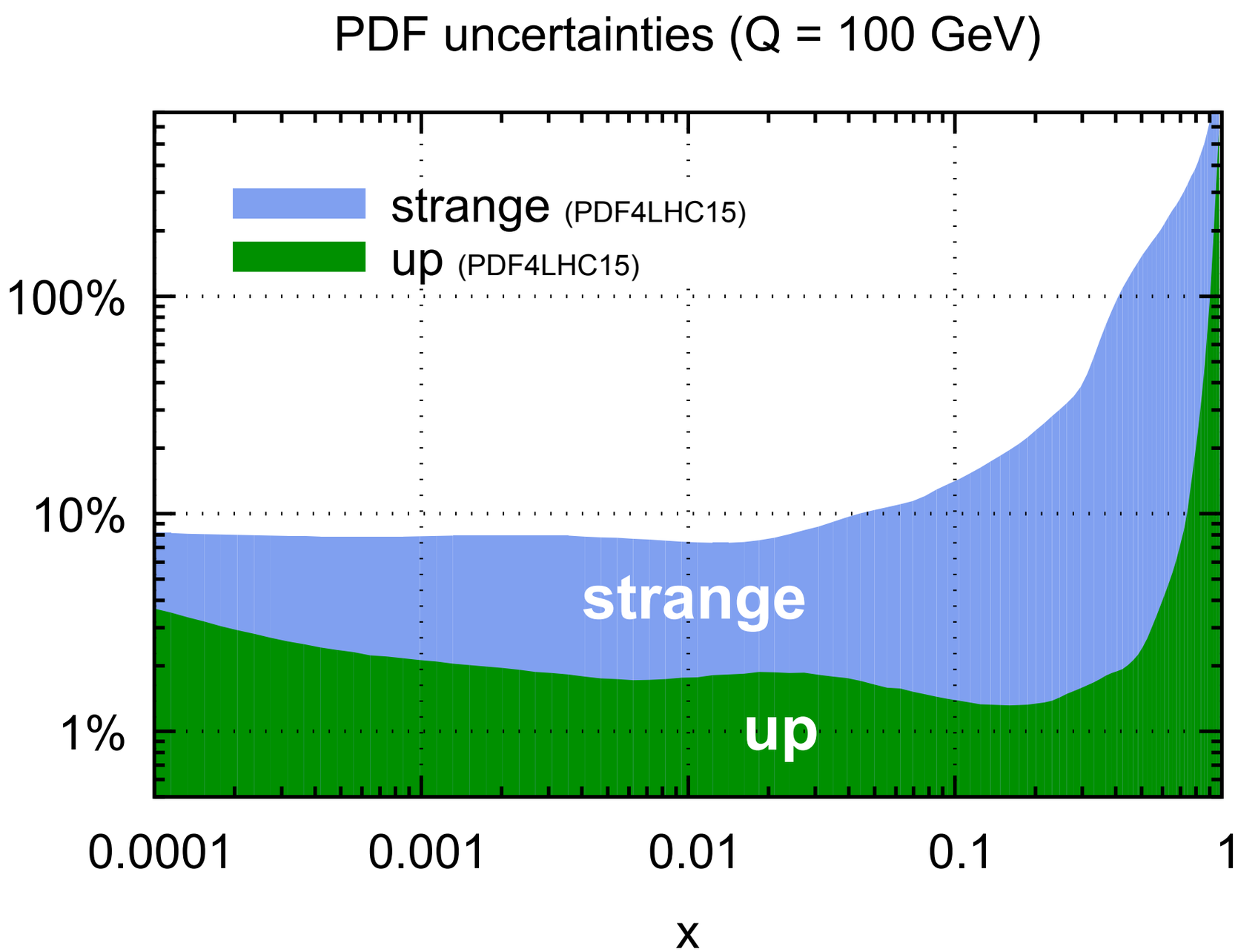
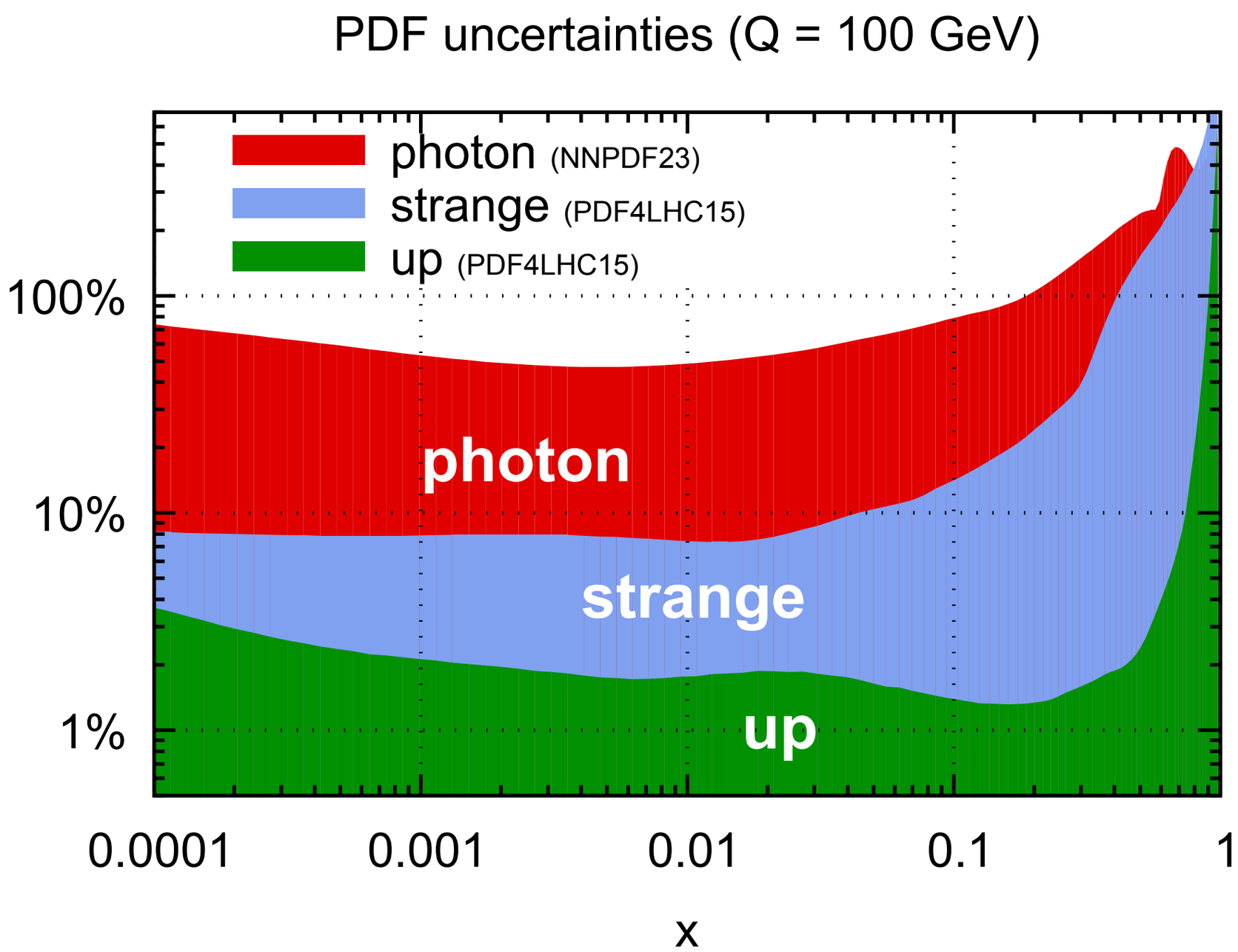
how well do we know the parton distributions?



- core partons (up, down, gluon) are quite well known



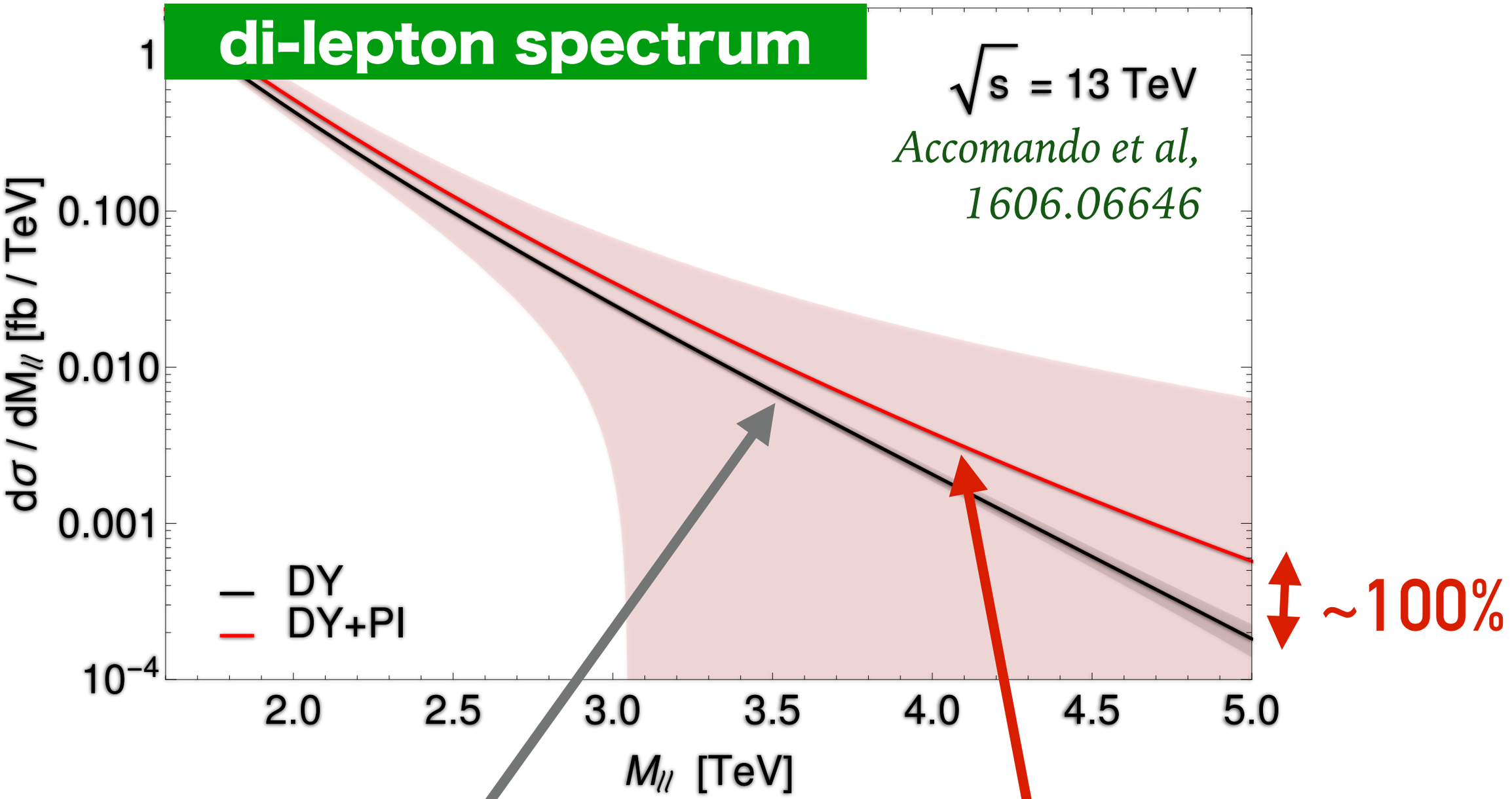
- core partons (up, down, gluon) are quite well known ~2%
- strangeness ~10%



- core partons (up, down, gluon) are quite well known ~2%
- strangeness ~10%

- one other parton, the **photon**, has been debated. The only model-independent determination (NNPDF23qed) has **O(100%) uncertainty**

IT MATTERS FOR DI-LEPTON, DI-BOSON, TTBAR, EW HIGGS, ETC.



normal DY contribution

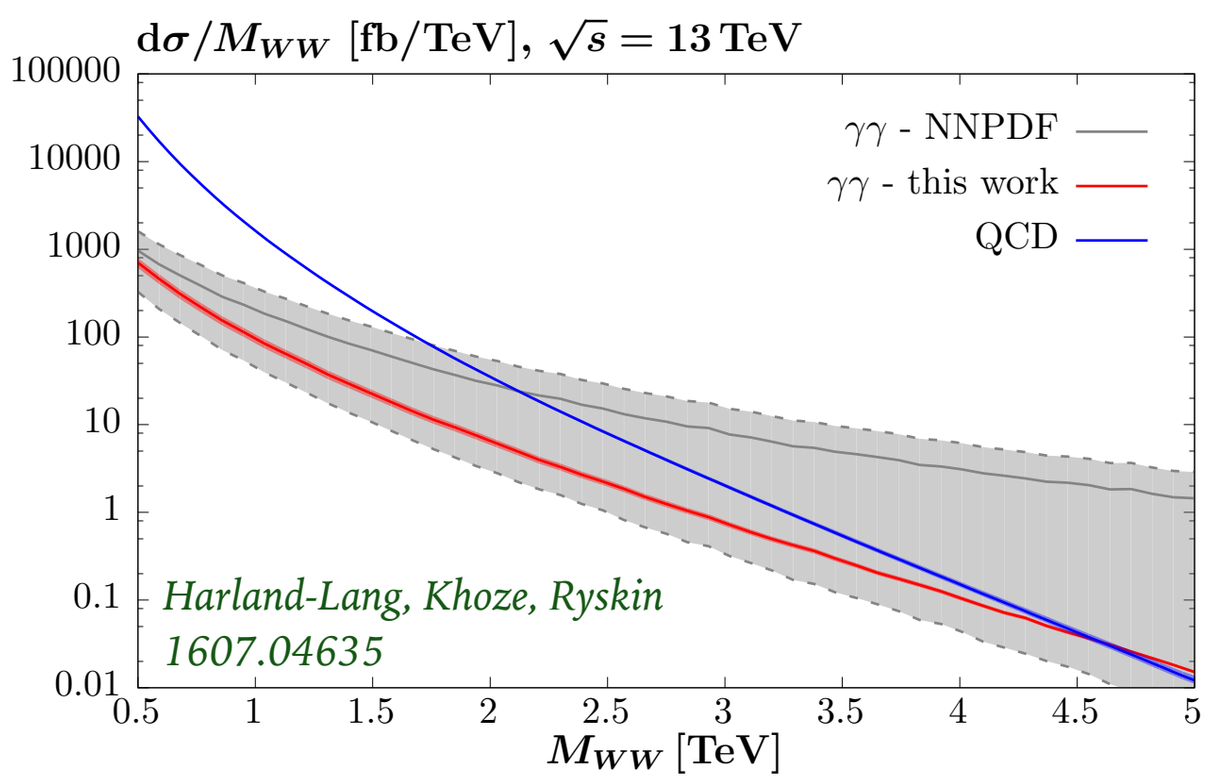
photon-induced contribution and uncertainty [NNPDF23]

where else does the photon come in?

- Electroweak corrections to almost any process
- Largest uncertainty on VBF Higgs and WH (±few %)

LHC-HXSWG YR4

- top production *Pagani, Tsinikos, Zaro, arXiv:1606.01915*
- VV production *1409.1803, 1510.08742, 1603.04874, 1601.07787, 1605.03419, 1604.04080,1607.04635, …*

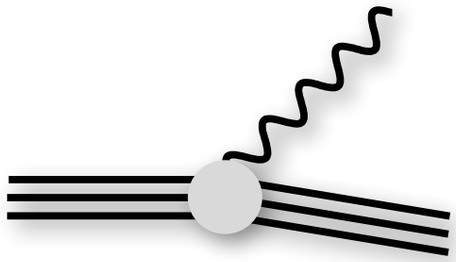


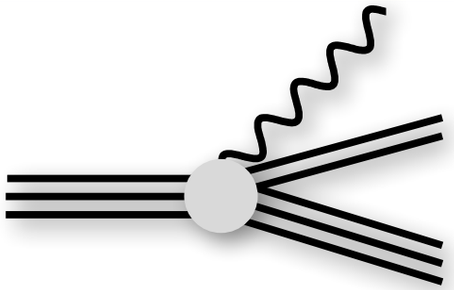
$\gamma\gamma$ (NNPDF) 100× larger than $q\bar{q}$

PHOTON PDF ESTIMATES (not exhaustive)

	elastic	inelastic	in LHAPDF?
Gluck Pisano Reya 2002	dipole	model	✘
MRST2004qed	✘	model	✓
NNPDF23qed	no separation; fit to data		✓
CT14qed	✘	model (data-constrained)	✓
CT14qed_inc	dipole	model (data-constrained)	✓
Martin Ryskin 2014	dipole (only electric part)	model	✘
Harland-Lang, Khoze Ryskin 2016	dipole	model	✘

elastic: Budnev, Ginzburg, Meledin, Serbo, 1975





YOU SHOULDN'T NEED A MODEL

ep scattering (i.e. structure functions) contains all info about proton's EM field

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study hypothetical ("BSM") heavy-neutral lepton production process

Calculate it in two ways

(1) in terms of structure functions (known)
(2) in terms of photon distribution (unknown)

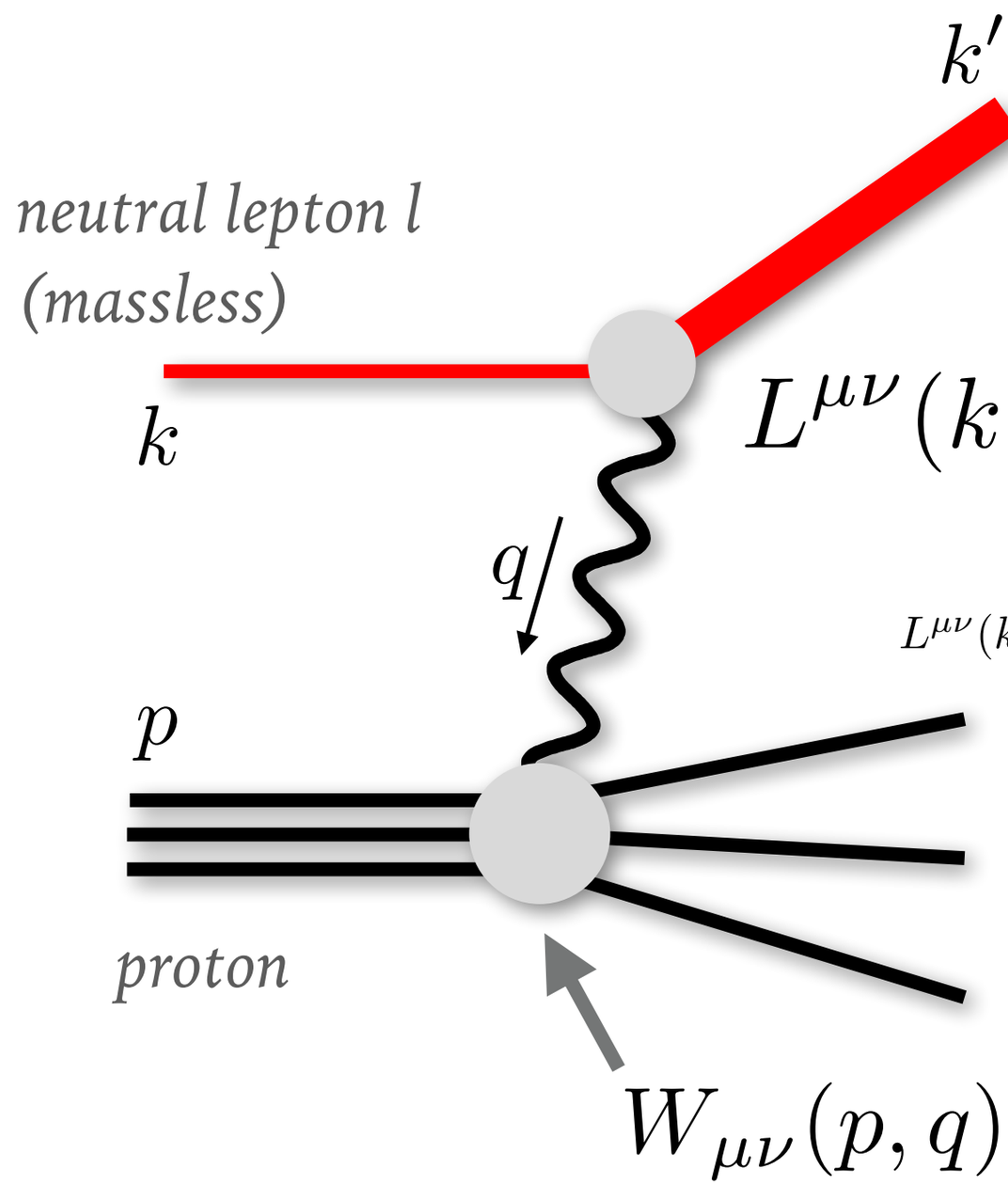
Equivalence gives us photon distirbution

Manohar, Nason, GPS & Zanderighi, arXiv:1607.04266
(inspired by Drees & Zeppenfeld, PRD39(1989)2536)

calculation

STEP 1

work out a cross section (exact) in terms of F_2 and F_L struct. fns.



$L^{\mu\nu}(k,q)$ *leptonic tensor, calculate with Feynman diag.*

$$\mathcal{L}_{\rm int} = (e/\Lambda)\overline{L}\,\sigma^{\mu\nu}F_{\mu\nu}\,l$$

$$L^{\mu\nu}(k,q) = \tfrac{1}{2}(e^2_{\rm ph}(q^2)/\Lambda^2){\rm Tr}\left(\not{k}'\left[\not{q},\gamma^\mu\right](\not{k}'+M)\left[\gamma^\nu,\not{q}\right]\right)$$

$$Q^2 = -q^2$$

$$x_{\rm Bj} = \frac{Q^2}{2p.q}$$

$$-g_{\mu\nu}F_1(x_{\rm Bj},Q^2) + \frac{p_\mu p_\nu}{p.q}F_2(x_{\rm Bj},Q^2) + \cdots$$

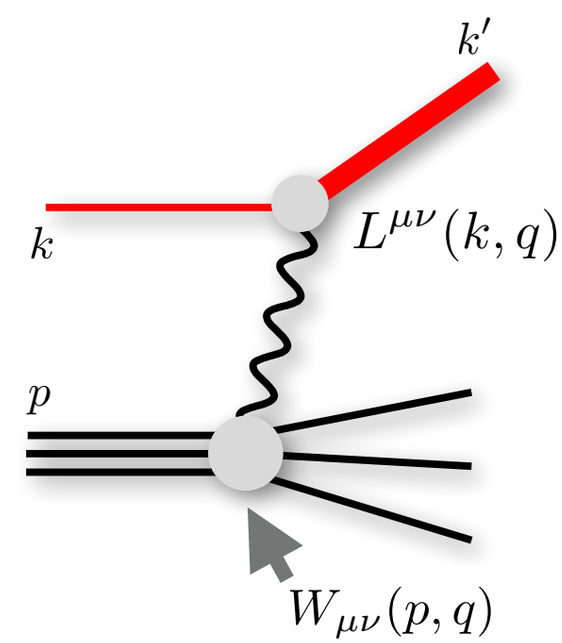
hadronic tensor, known in terms of F_2 and F_L

$$\sigma = \frac{1}{4p\cdot k}\int\frac{d^4q}{(2\pi)^4 q^4}e^2_{\rm ph}(q^2)\left[4\pi W_{\mu\nu}\,L^{\mu\nu}(k,q)\right]\times 2\pi\delta((k-q)^2-M^2)$$

Cross section in terms of structure functions

- Lagrangian of interaction: (magnetic moment coupling) $\mathcal{L}_{\rm int} = (e/\Lambda)\overline{L}\,\sigma^{\mu\nu}F_{\mu\nu}\,l$
- Using leptons neutral and taking Λ large, ensure that only single-photon exchange is relevant
- **Answer is exact up to $1/\Lambda$ corrections**

$$\sigma = \frac{c_0}{2\pi}\int_x^{1-\frac{2xm_p}{M}}\frac{dz}{z}\int_{Q^2_{\rm min}}^{Q^2_{\rm max}}\frac{dQ^2}{Q^2}\alpha^2_{\rm ph}(-Q^2)\Bigg[\Bigg(2-2z+z^2 + \frac{2x^2m_p^2}{Q^2}+\frac{z^2Q^2}{M^2}-\frac{2zQ^2}{M^2}-\frac{2x^2Q^2m_p^2}{M^4}\Bigg)F_2(x/z,Q^2) + \left(-z^2-\frac{z^2Q^2}{2M^2}+\frac{z^2Q^4}{2M^4}\right)F_L(x/z,Q^2)\Bigg]$$



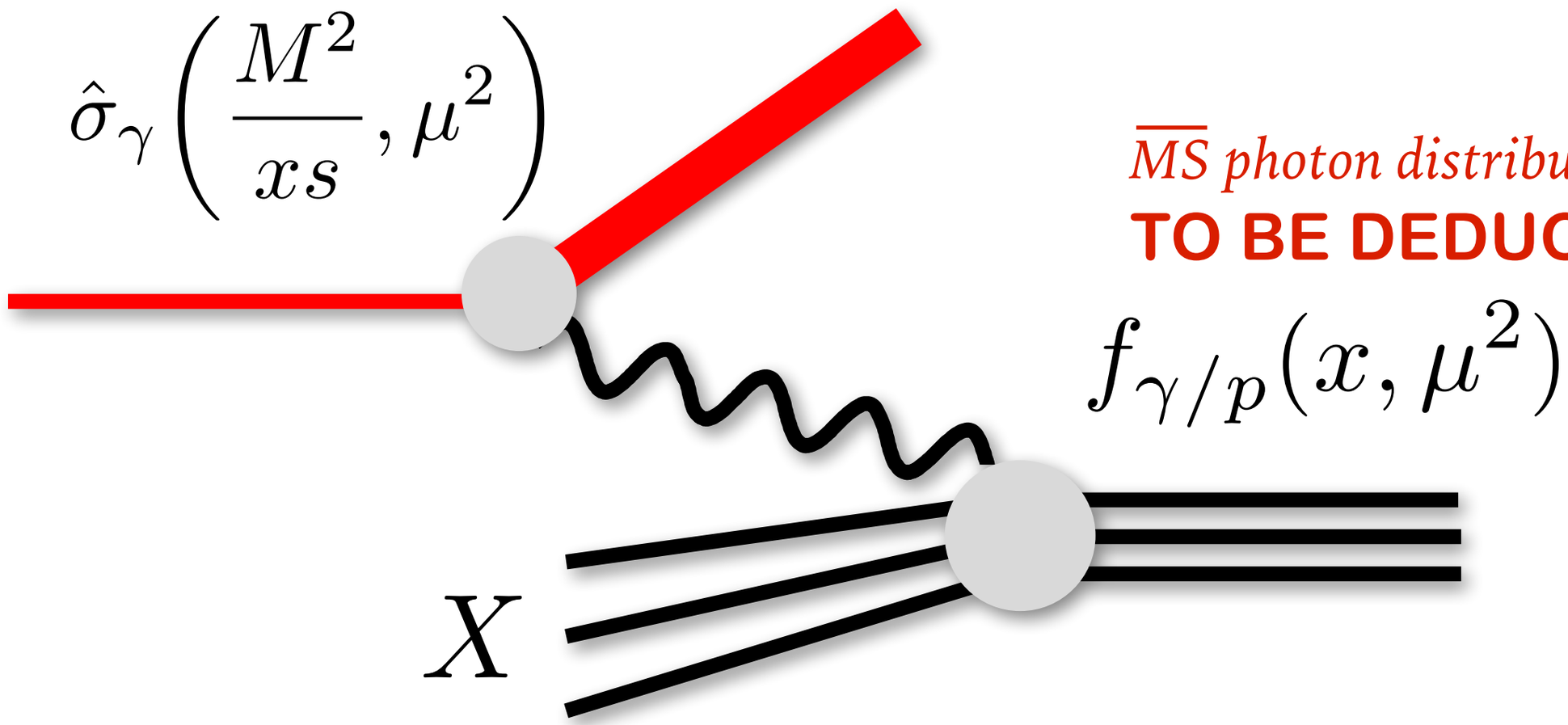
$$c_0 = 16\pi^2/\Lambda^2$$

STEP 2

work out same cross section in terms of a photon distribution

hard-scattering cross section
calculate in collinear factorisation

$$\hat{\sigma}_\gamma\left(\frac{M^2}{xs}, \mu^2\right)$$



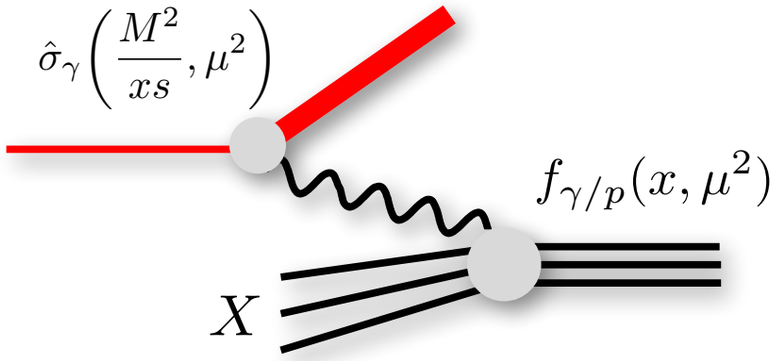
$\overline{MS}$ photon distribution:

TO BE DEDUCED

$$f_{\gamma/p}(x, \mu^2)$$

$$\sigma = c_0 \sum_a \int \frac{dx}{x}\, \hat{\sigma}_a\left(\frac{M^2}{xs}, \mu^2\right)\, x f_{a/p}\left(x, \mu^2\right)$$

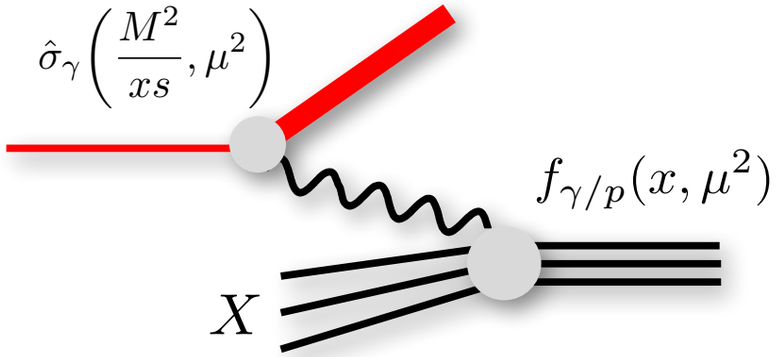
Cross section in terms of structure functions



- Hard cross section driven by the photon distribution at LO

$$\hat{\sigma}_a(z, \mu^2) = \alpha(\mu^2)\delta(1-z)\delta_{a\gamma}$$

Cross section in terms of structure functions



- Hard cross section driven by the photon distribution at LO

$$\hat{\sigma}_a(z, \mu^2) = \alpha(\mu^2)\delta(1-z)\delta_{a\gamma} + \frac{\alpha^2(\mu^2)}{2\pi}\Bigg[-2 + 3z + z p_{\gamma q}(z) \ln \frac{M^2(1-z)^2}{z\mu^2}\Bigg] \sum_{i \in \{q, \bar{q}\}} e_i^2 \delta_{ai} + \ldots$$

- Quarks and gluons come in at higher orders

ACCURACY AIM

- Take quark and gluon distributions $\sim O(1)$
- α is QED coupling, α_s is QCD coupling, $L = \ln \mu^2/m_p{}^2$
  - Take $L \sim 1/\alpha_s$, so all $(\alpha_s L)^n \sim 1$
  - Think of $\alpha \sim (\alpha_s)^2$
- To first order, photon distribution $\sim (\alpha\, L)$
- we aim to control all terms:
  - $\alpha\, L\, (\alpha_s L)^n$ [LO]
  - $\alpha_s\, \alpha\, L\, (\alpha_s L)^n \equiv \alpha\, (\alpha_s L)^n$ [NLO — extra α_s or $1/L$]
  - $\alpha^2\, L^2\, (\alpha_s L)^n$ [NLO — extra $\alpha\, L$]
- Matching done at large M^2 and μ^2 to eliminate higher twists

STEP 3
equate them to deduce the photon distribution (LUXqed)

$$x f_{\gamma/p}(x,\mu^2) = \frac{1}{2\pi\alpha(\mu^2)}\int_x^1 \frac{dz}{z}\left\{\int_{\frac{x^2 m_p^2}{1-z}}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2}\alpha^2(Q^2)\right.$$

$$\left[\left(zp_{\gamma q}(z) + \frac{2x^2 m_p^2}{Q^2}\right)F_2(x/z,Q^2) - z^2 F_L\left(\frac{x}{z},Q^2\right)\right]$$

$$\left. - \alpha^2(\mu^2) z^2 F_2\left(\frac{x}{z},\mu^2\right)\right\}$$

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with $F_2 \sim \sum_q e_q^2 x q(x)$ this is just (LO) DGLAP-like piece

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$$x f_{\gamma/p}(x,\mu^2) = \frac{1}{2\pi\alpha(\mu^2)} \int_x^1 \frac{dz}{z} \left\{ \int_{\frac{x^2 m_p^2}{1-z}}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2} \alpha^2(Q^2) \left[\left(z p_{\gamma q}(z) + \frac{2x^2 m_p^2}{Q^2} \right) F_2(x/z, Q^2) - z^2 F_L\left(\frac{x}{z}, Q^2\right) \right] - \alpha^2(\mu^2) z^2 F_2\left(\frac{x}{z}, \mu^2\right) \right\}$$

At low Q^2, F_2 and F_L come directly from data (non.pert.)
At high Q^2, get them from PDFs, including $O(\alpha_s)$ (NLO) terms

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Terms at boundaries are suppresed by $1/L$ (NLO)

STEP 3
equate them to deduce the photon distribution (LUXqed)

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terms at boundary $\sim \mu^2$ ensure $\overline{\text{MS}}$ fact. scheme

STEP 3
equate them to deduce the photon distribution (LUXqed)

$$x f_{\gamma/p}(x,\mu^2) = \frac{1}{2\pi\alpha(\mu^2)} \int_x^1 \frac{dz}{z} \left\{ \int_{\frac{x^2 m_p^2}{1-z}}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2} \alpha^2(Q^2) \left[\left(z p_{\gamma q}(z) + \frac{2x^2 m_p^2}{Q^2} \right) F_2(x/z, Q^2) - z^2 F_L\left(\frac{x}{z}, Q^2\right) \right] - \alpha^2(\mu^2) z^2 F_2\left(\frac{x}{z}, \mu^2\right) \right\}$$

QED running of α accounts for most $(\alpha L)^2$ effects (NLO)
(others come in the way we match to normal PDFs)

cross-checks

Cross checks & literature comparisons

- Repeat calculation with for a different process (γp→H+X, via γγ→H). Intermediate results differ, **final photon distribution is identical**.
- Substitute elastic-scattering component of F_2 and F_L:

$$F_2^{\rm el} = \frac{[G_E(Q^2)]^2 + [G_M(Q^2)]^2\tau}{1+\tau}\delta(1-x)\,,$$

$$F_L^{\rm el} = \frac{[G_E(Q^2)]^2}{\tau}\delta(1-x)\,, \qquad \tau \;=\; Q^2/(4m_p^2)$$

and reproduce widely-used **Equivalent Photon Approximation** with electric (G_E) and magnetic (G_M) Sachs proton form factors

Budnev et al., Phys.Rept.15(1975)181

Cross checks & literature comparisons

- A core part of our answer

$$\left[\left(zp_{\gamma q}(z)+\frac{2x^2m_p^2}{Q^2}\right)F_2(x/z,Q^2)-z^2F_L\left(\frac{x}{z},Q^2\right)\right]$$

appears in literature for **QED compton process ep → eγX** (but with inexact treatment of the upper and lower limits for Q^2 integration)

Anlauf et. al, CPC70(1992)97
Mukherjee & Pisano, hep-ph/0306275

- [NB other literature has expression for photon distribution in terms of F_2 and F_1 that doesn't reproduce DGLAP limit]

Luszczak, Schäfer & Szczurek, arXiv:1510.00294

Cross checks & literature comparisons

- μ^2 derivative of our answer should reproduce known DGLAP QCD-QED splitting functions
- At LO, this is trivial.
- At NLO we get relations between QED-QCD splitting functions (P) and DIS coefficient functions (C)

$$P_{\gamma q}^{(1,1)} = e_q^2 \left[p_{\gamma q} \otimes C_{2q} - h \otimes C_{Lq} + (\bar{p}_{\gamma q} - h) \otimes P_{qq}^{(1,0)} \right] ,$$

$$P_{\gamma g}^{(1,1)} = \sum_{q,\bar{q}} e_q^2 \left[p_{\gamma q} \otimes C_{2g} - h \otimes C_{Lg} + (\bar{p}_{\gamma q} - h) \otimes P_{qg}^{(1,0)} \right] ,$$

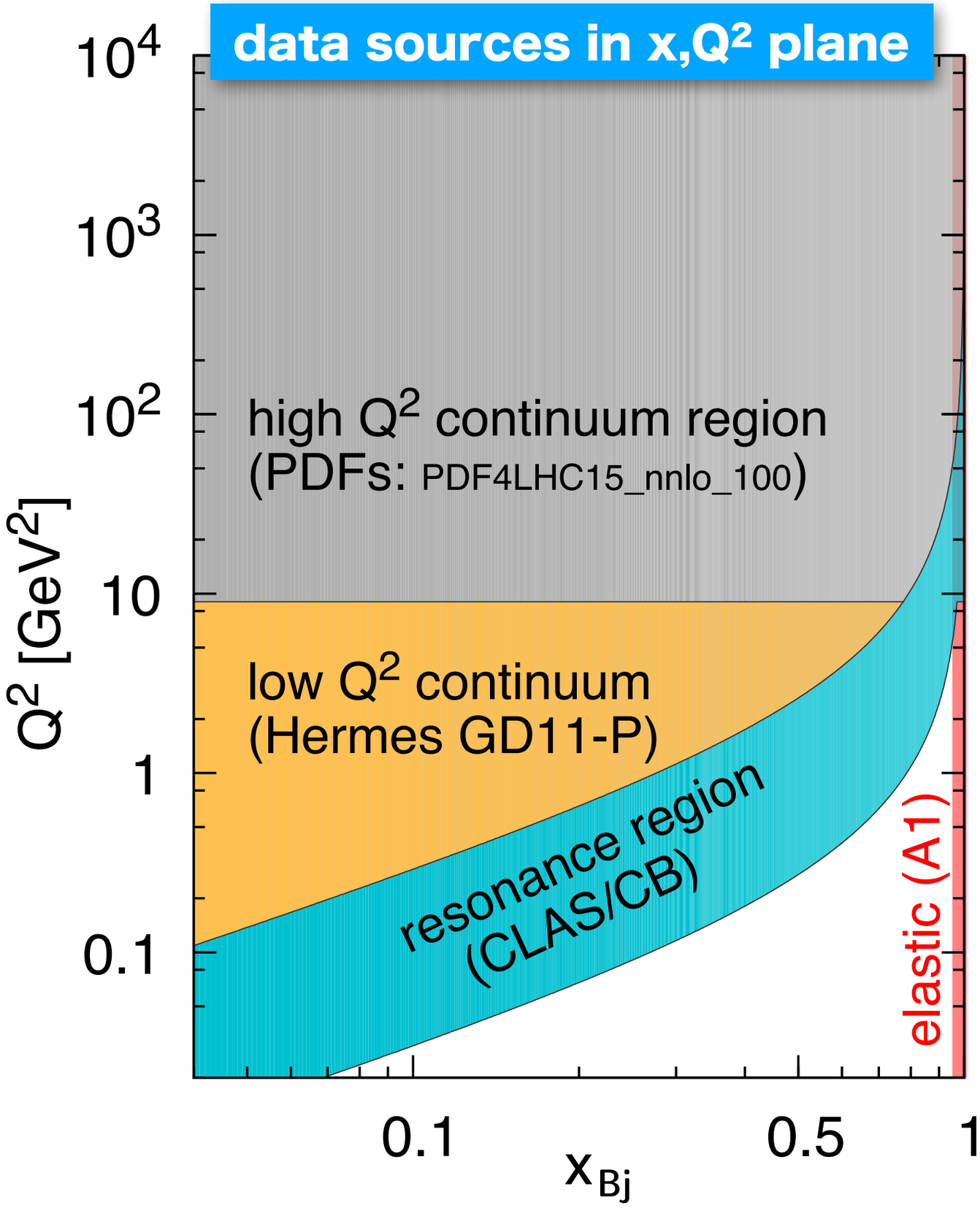
$$P_{\gamma\gamma}^{(1,1)} = (2\pi)^2 b_\alpha^{(1,2)} \delta(1-x) = -C_F N_C \sum_q e_q^2 \delta(1-x)$$

$h(z) \equiv z$ and $\bar{p}_{\gamma q}(z) \equiv p_{\gamma q}(z) \ln \frac{1}{1-z}$

- These **agree with de Florian, Sborlini & Rodrigo results**

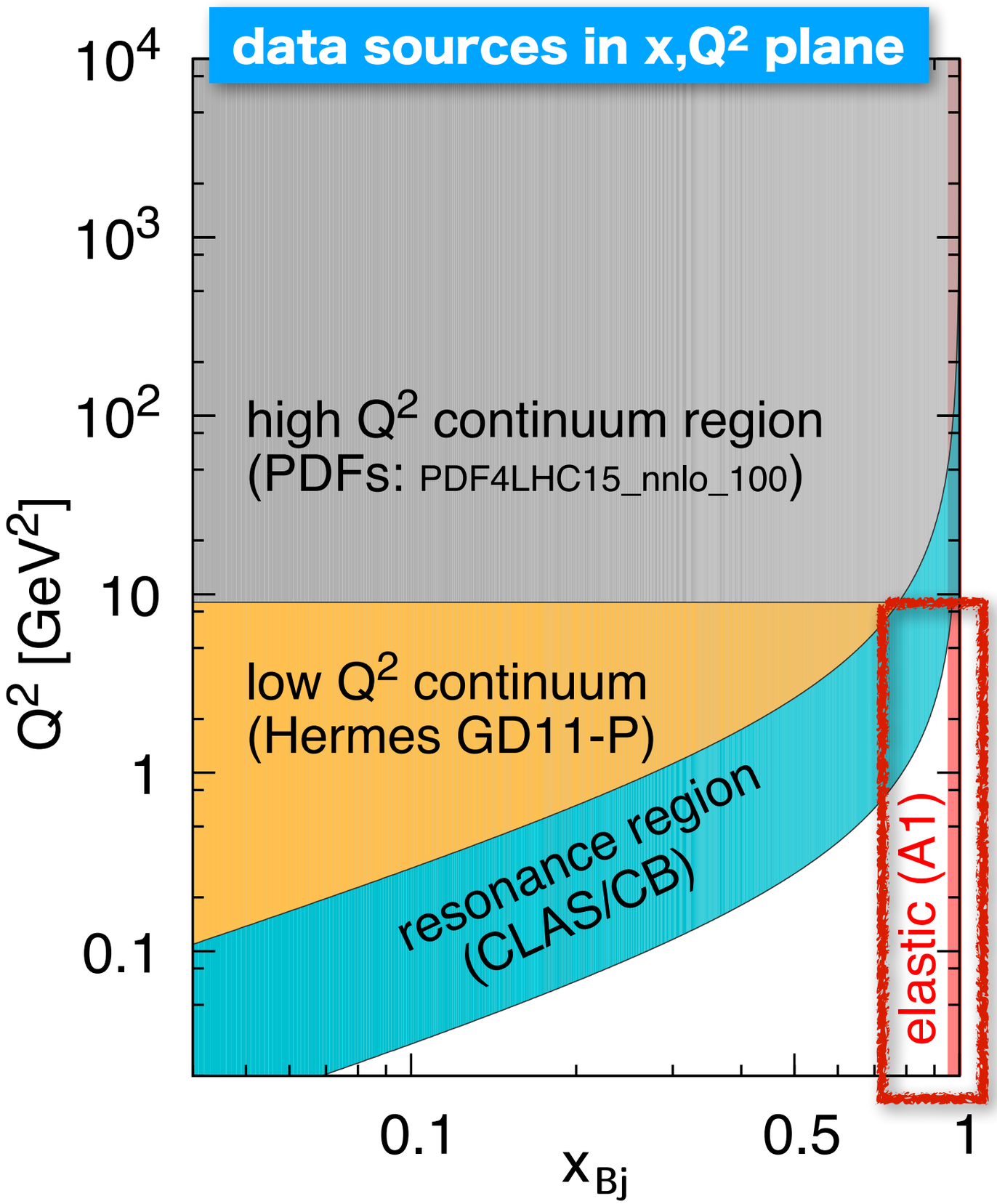
for $O(\alpha\, \alpha_s)$ terms, arXiv:1512.00612

data inputs

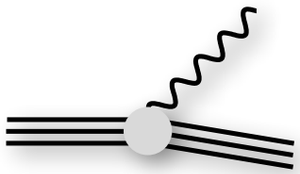


DATA

- x, Q^2 plane naturally breaks up into regions with different physical behaviours and data sources
- We don't use F_2 and F_L data directly, but various rather fits to data



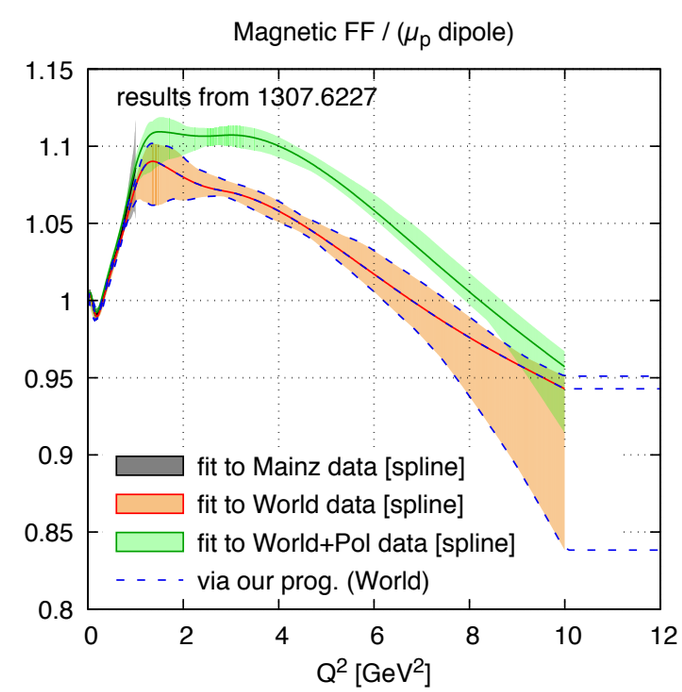
ELASTIC COMPONENT

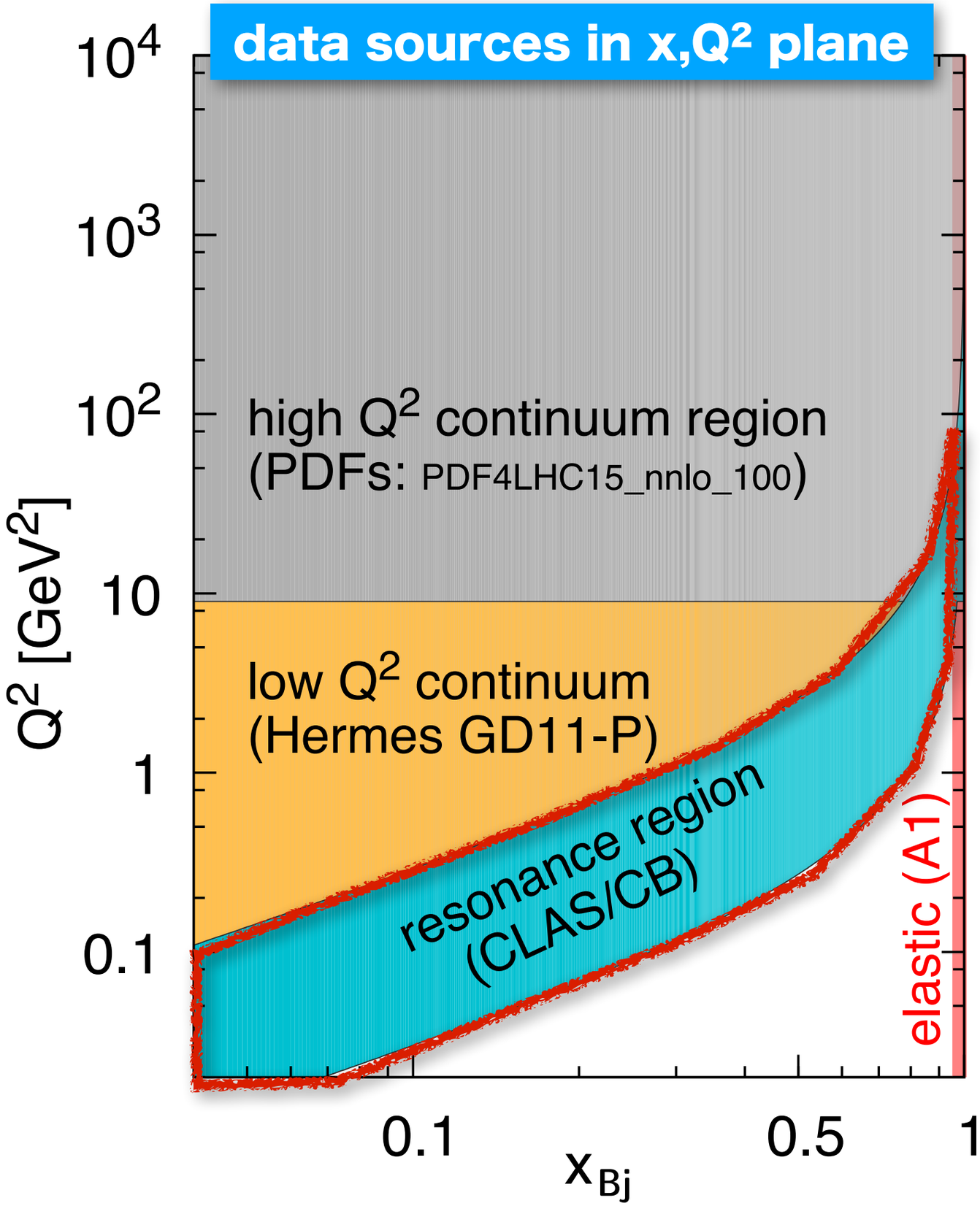


- Elastic component of $F_{2/L}$ lives at $x=1$
- Express in terms of Sachs Form factors

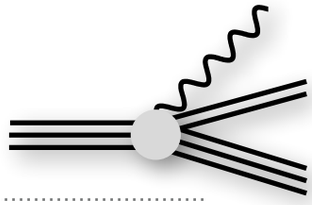
$$F_2^{\rm el} = \frac{[G_E(Q^2)]^2 + [G_M(Q^2)]^2\tau}{1+\tau}\delta(1-x)\,,$$

$$F_L^{\rm el} = \frac{[G_E(Q^2)]^2}{\tau}\delta(1-x)\,,\quad \tau = Q^2/(4m_p^2)$$

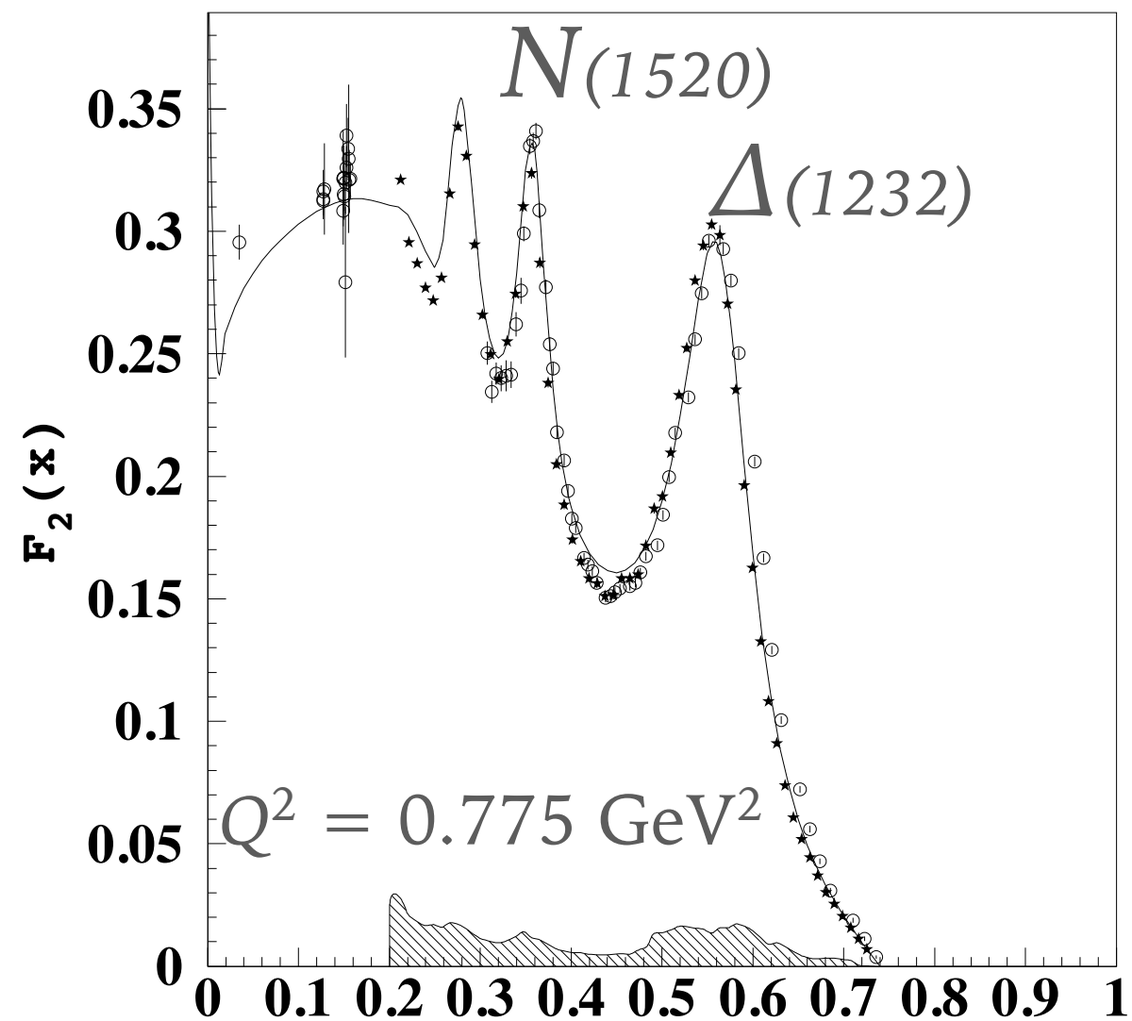


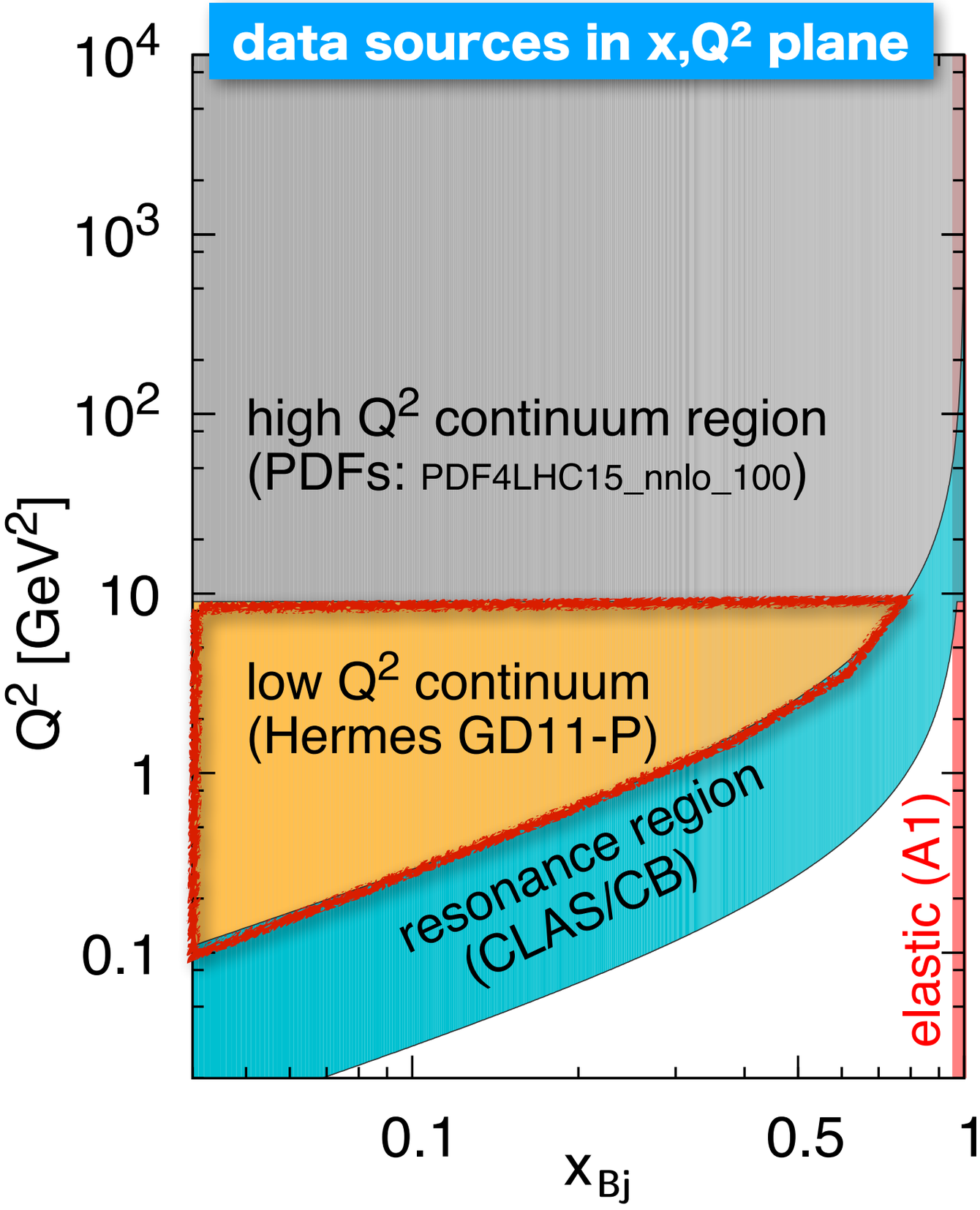


RESONANCE COMPONENT



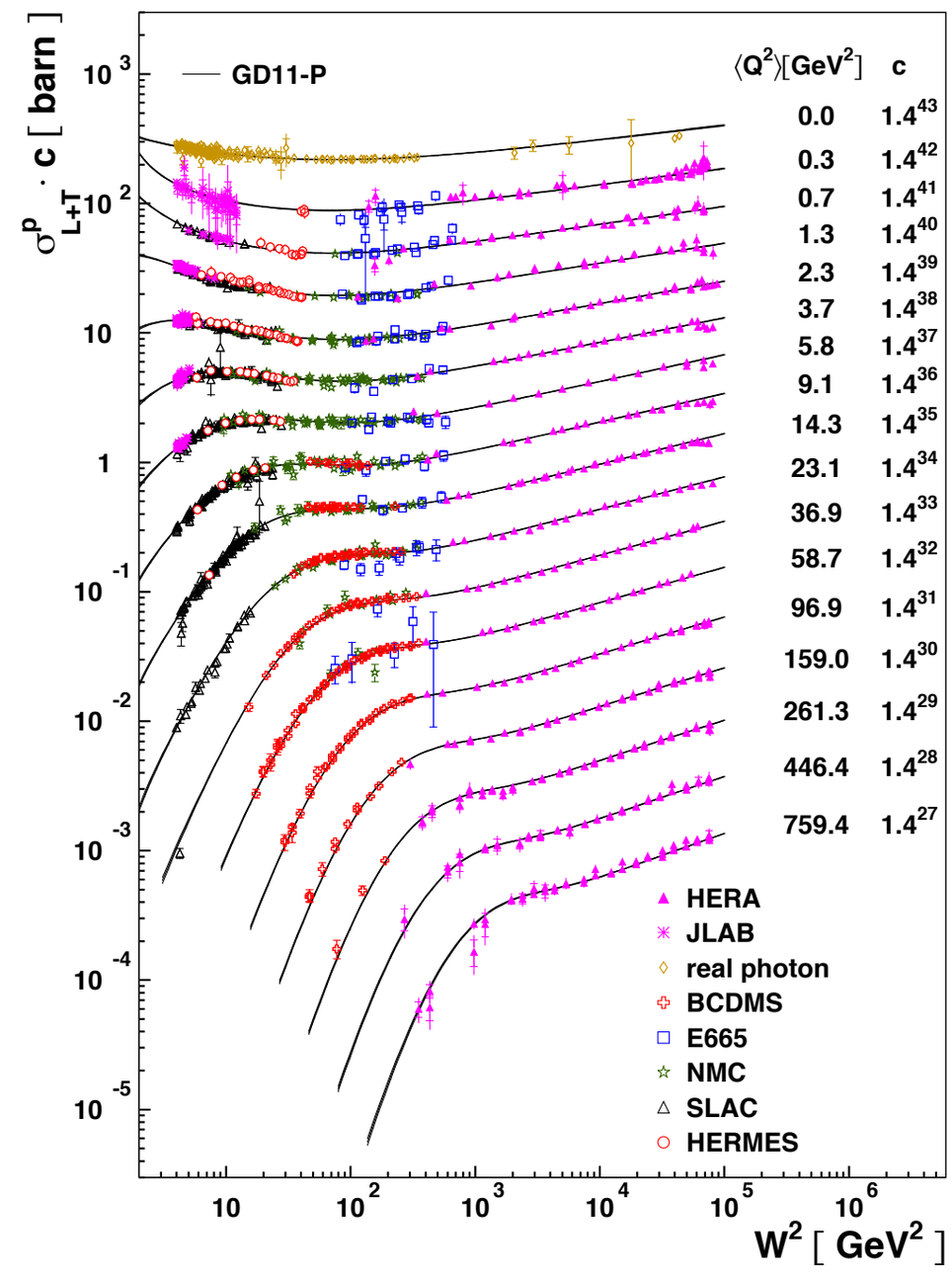
- proton gets excited, e.g. to $\Delta \rightarrow p\pi$ and higher resonances
- relevant for $(m_p+m_\pi)^2 < W^2 < 3.5\,\mathrm{GeV}^2$

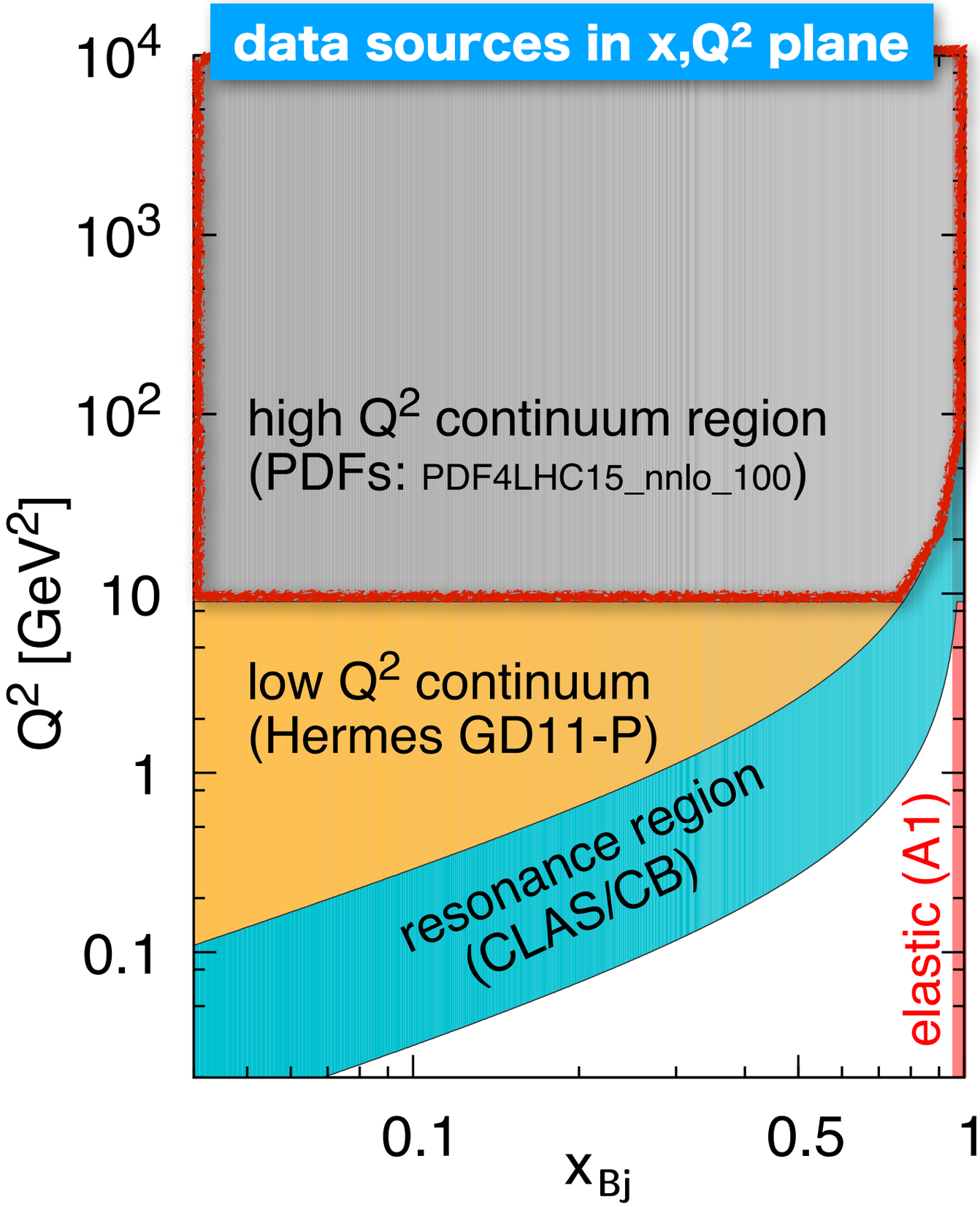




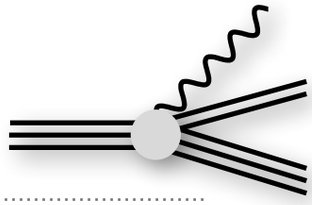
CONTINUUM COMPONENT

- Much data
- For $Q^2 \to 0$, $\sigma_{\gamma p}$ indep. of Q^2 at fixed W^2

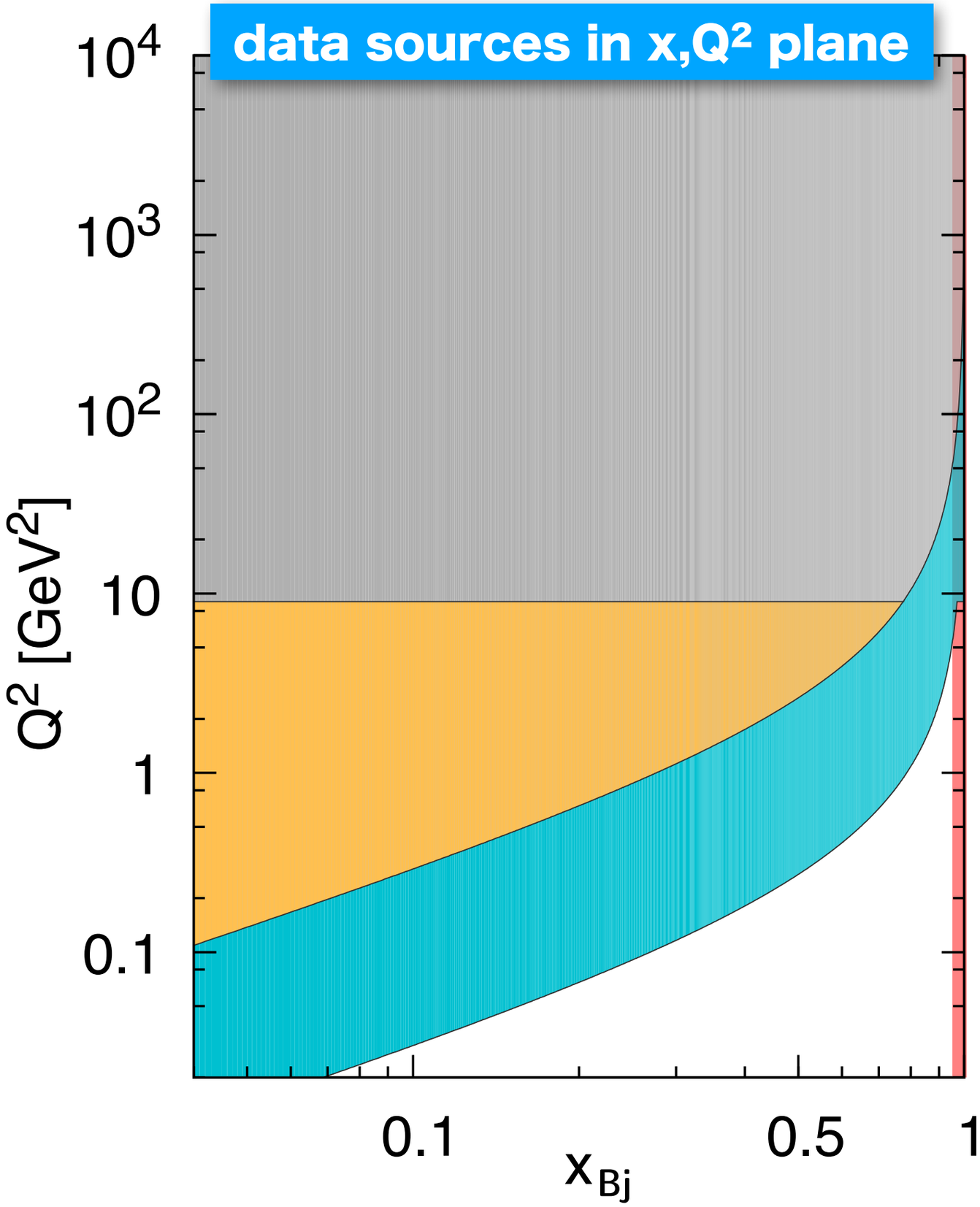




CONTINUUM COMPONENT



- Less data direct data for F_2 and F_L at high Q^2
- But we can reliably use PDFs and coefficient functions (up to NNLO) to calculate them
- Our default choice is PDF4LHC15_nnlo_100 (and zero-mass variable flavour-number scheme)



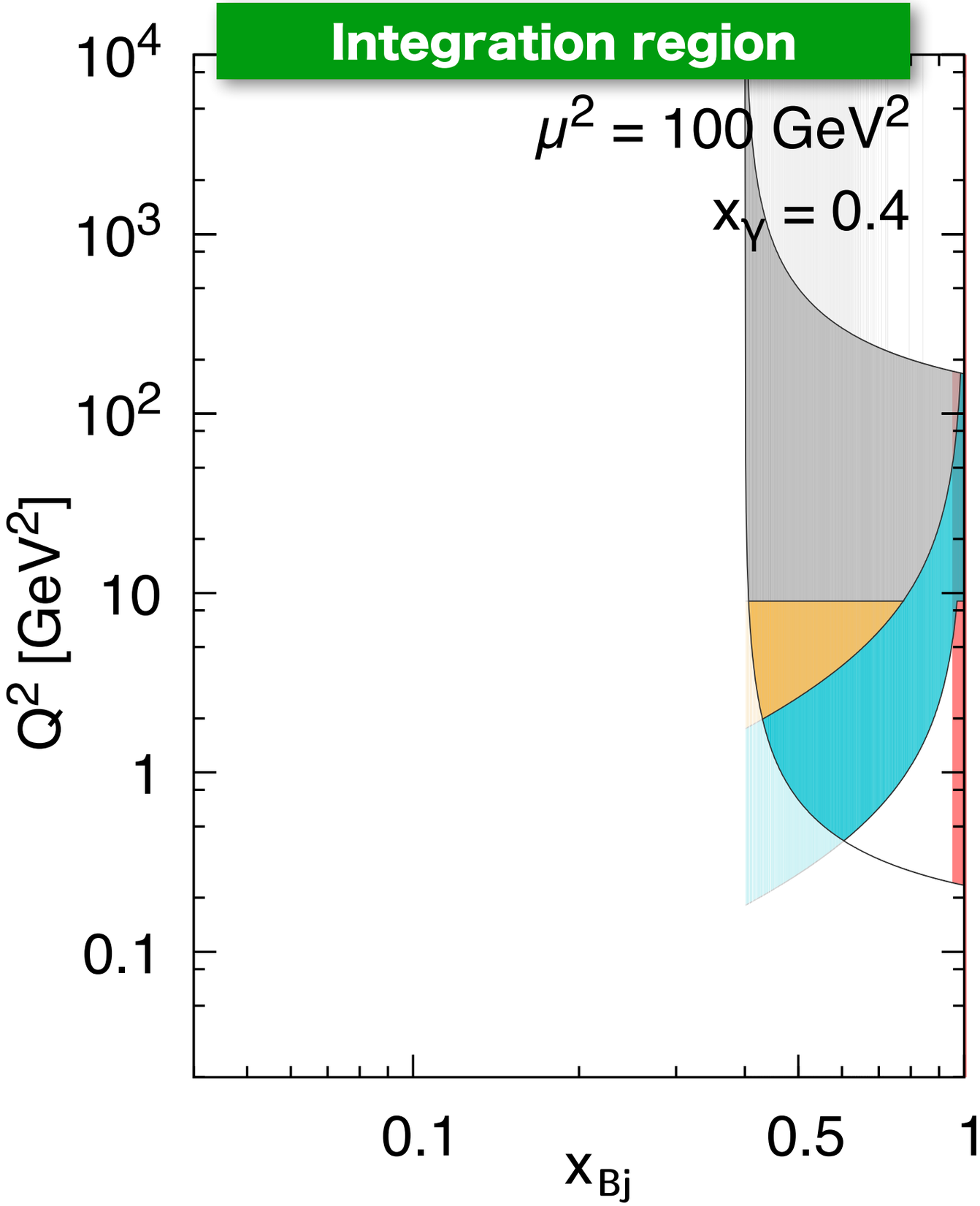
INTEGRATION REGION

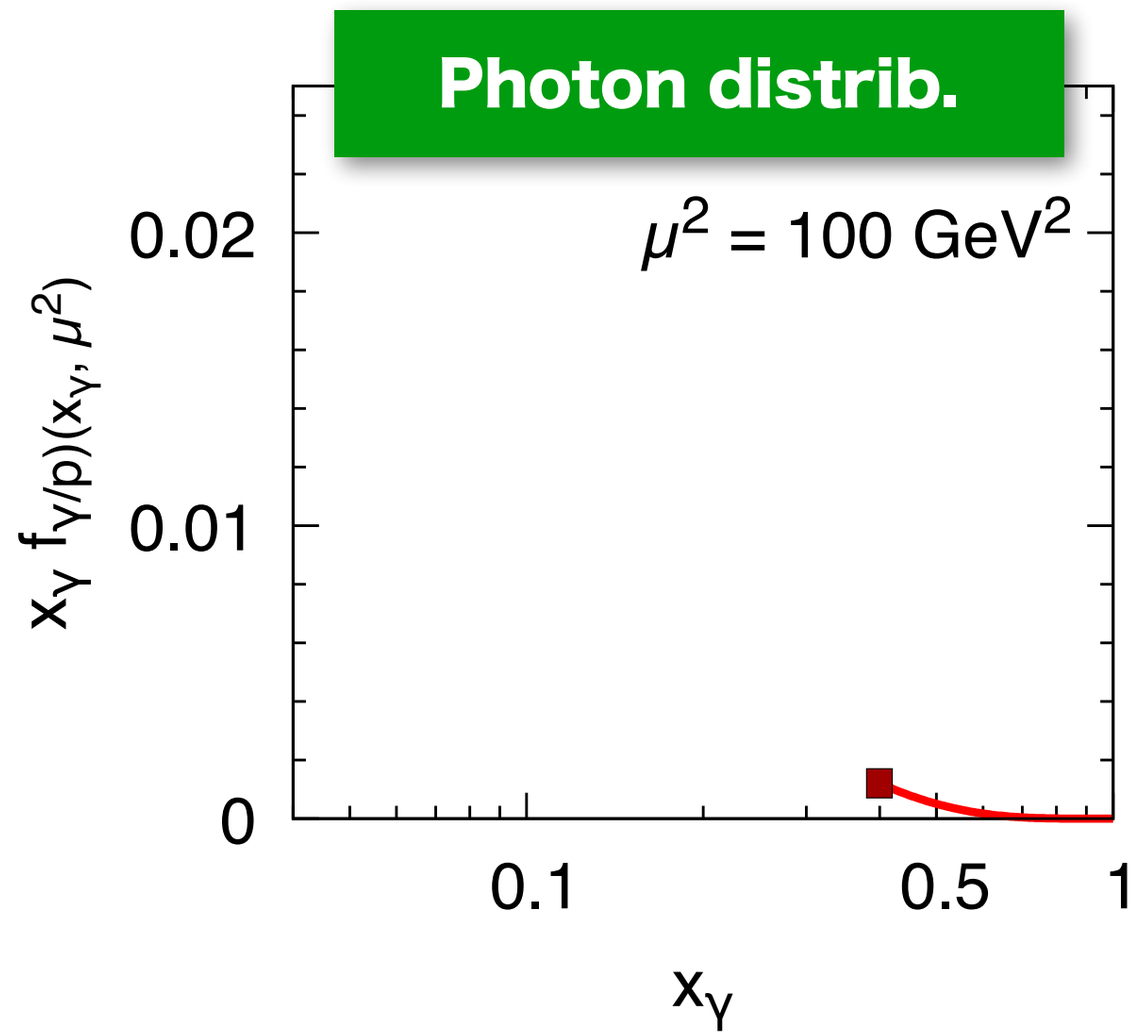
- depends on momentum fraction of the photon (x_γ) and factorisation scale (μ^2)

$$x f_{\gamma/p}(x,\mu^2) = \frac{1}{2\pi\alpha(\mu^2)} \int_x^1 \frac{dz}{z} \Bigg\{ \int_{\frac{x^2 m_p^2}{1-z}}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2} \alpha^2(Q^2) \left[\left(z p_{\gamma q}(z) + \frac{2x^2 m_p^2}{Q^2} \right) F_2(x/z, Q^2) - z^2 F_L\left(\frac{x}{z}, Q^2\right) \right] - \alpha^2(\mu^2) z^2 F_2\left(\frac{x}{z}, \mu^2\right) \Bigg\}, \quad (6)$$

INTEGRATION REGION

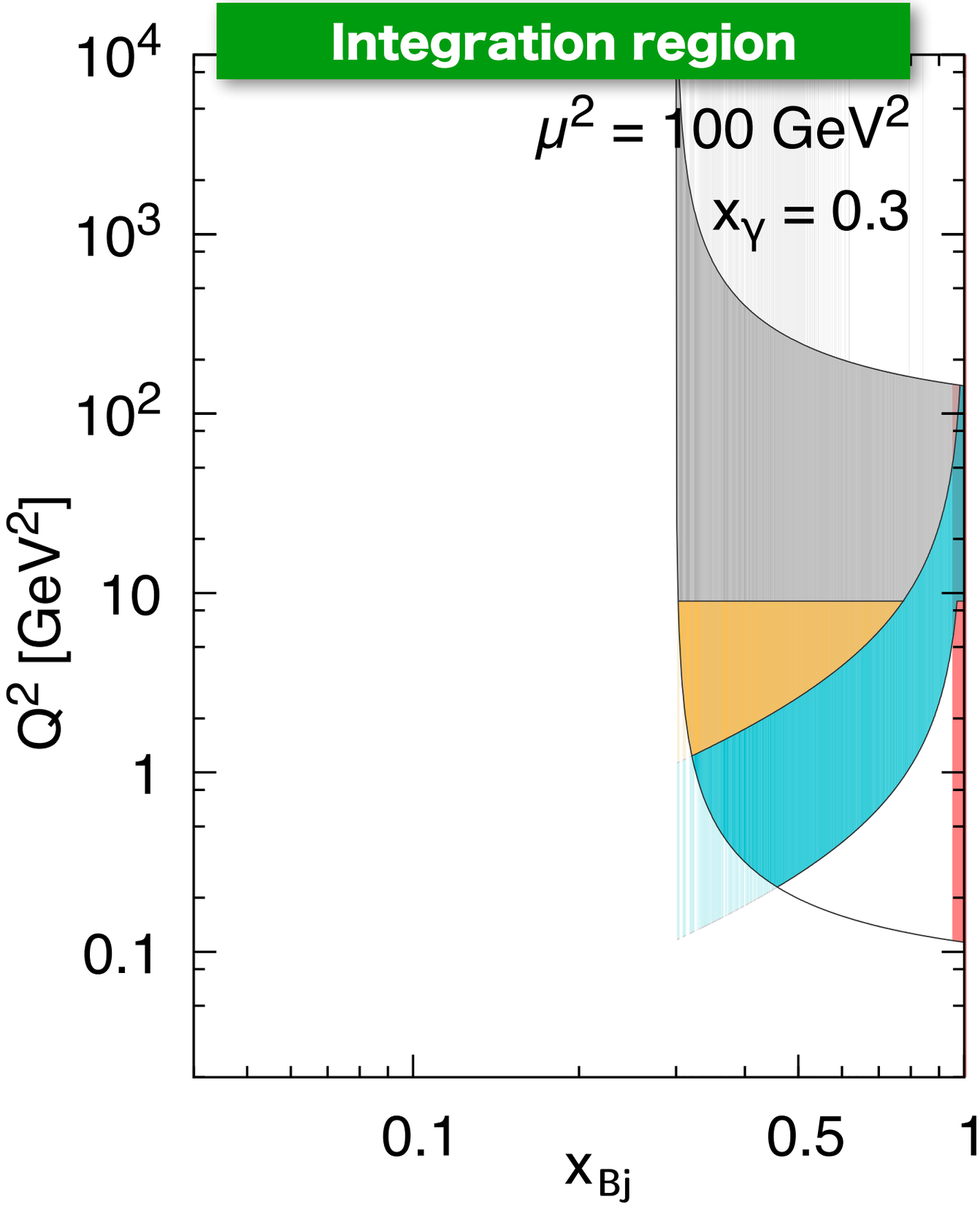
- depends on momentum fraction of the photon (x_γ) and factorisation scale (μ^2)

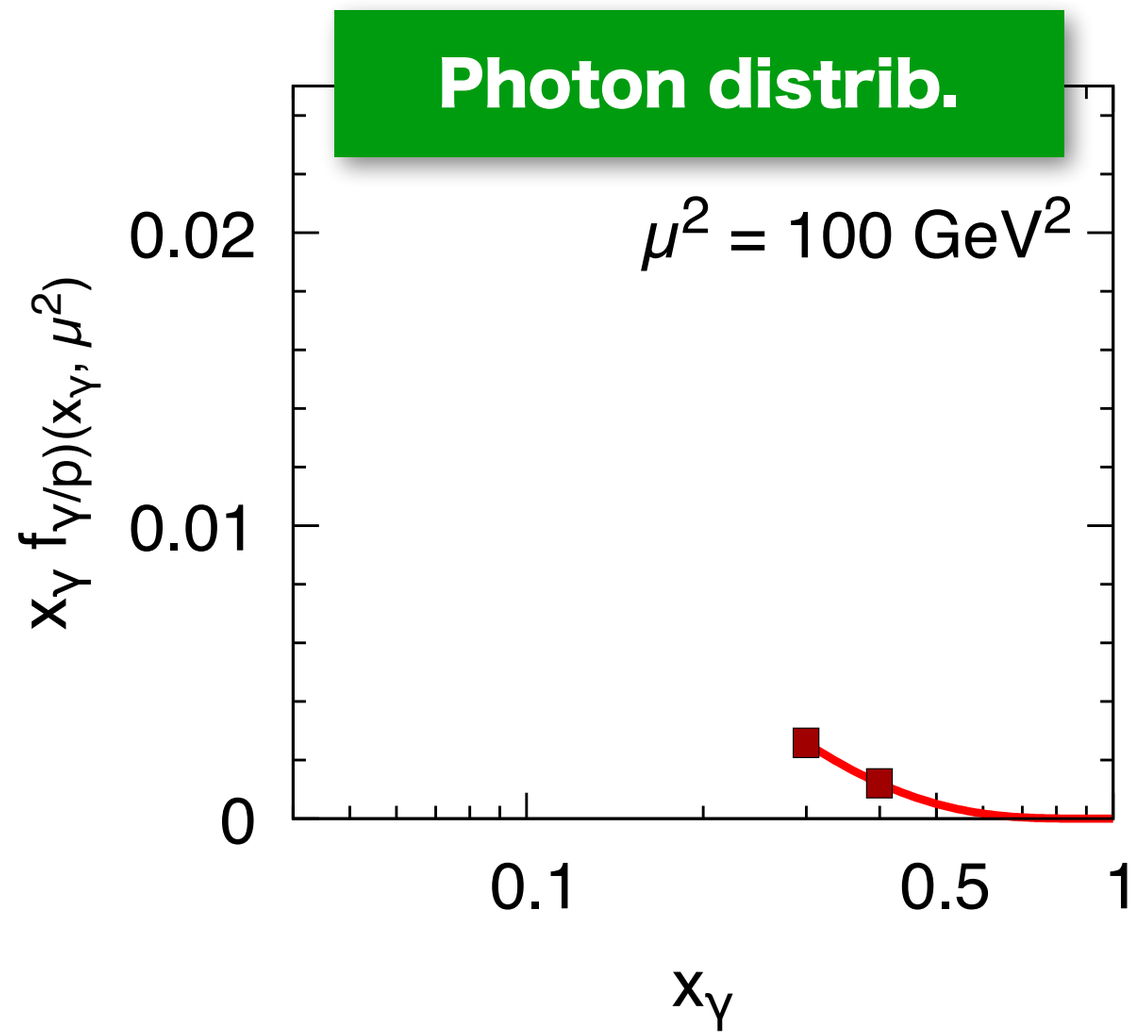


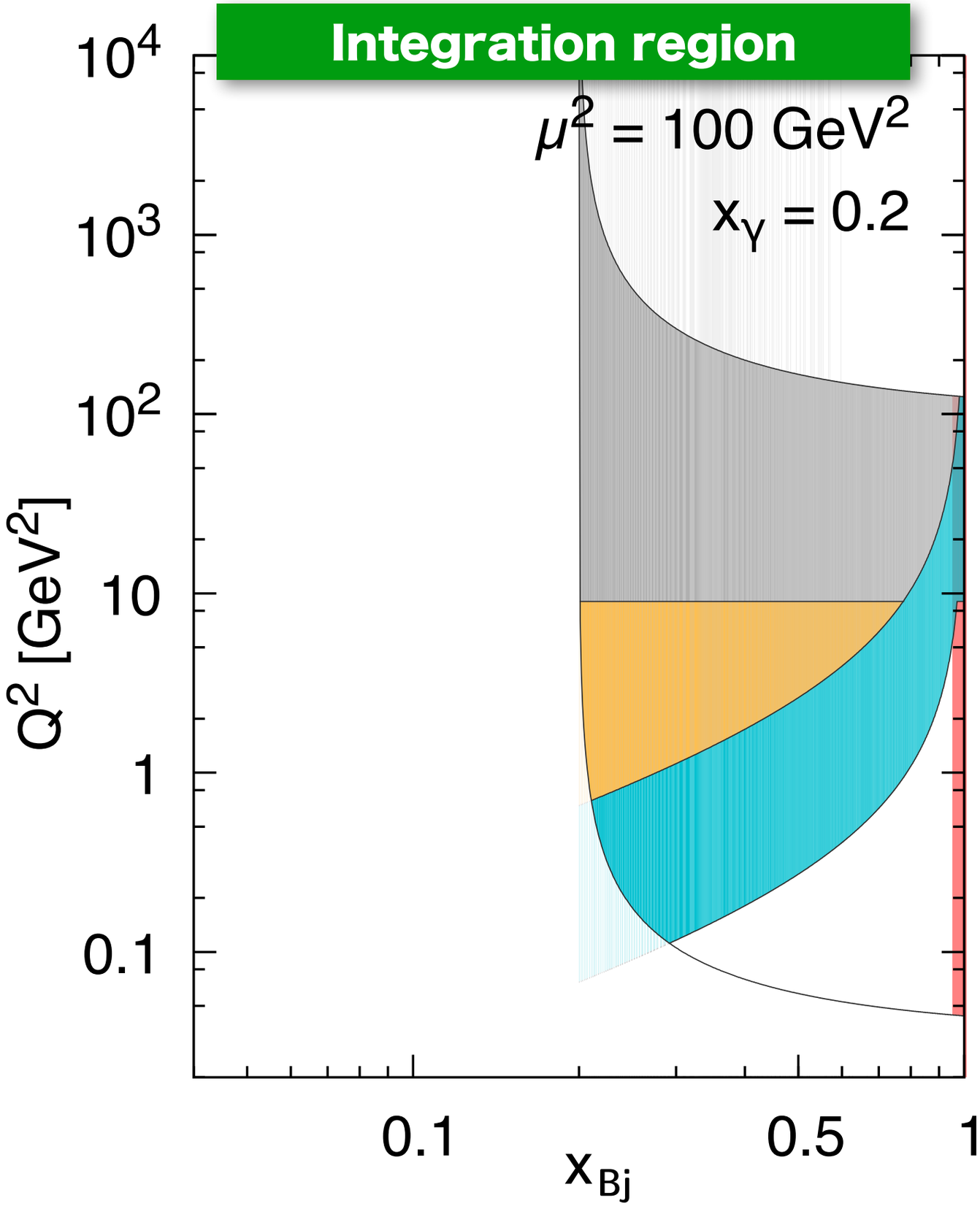


INTEGRATION REGION

- depends on momentum fraction of the photon (x_γ) and factorisation scale (μ^2)

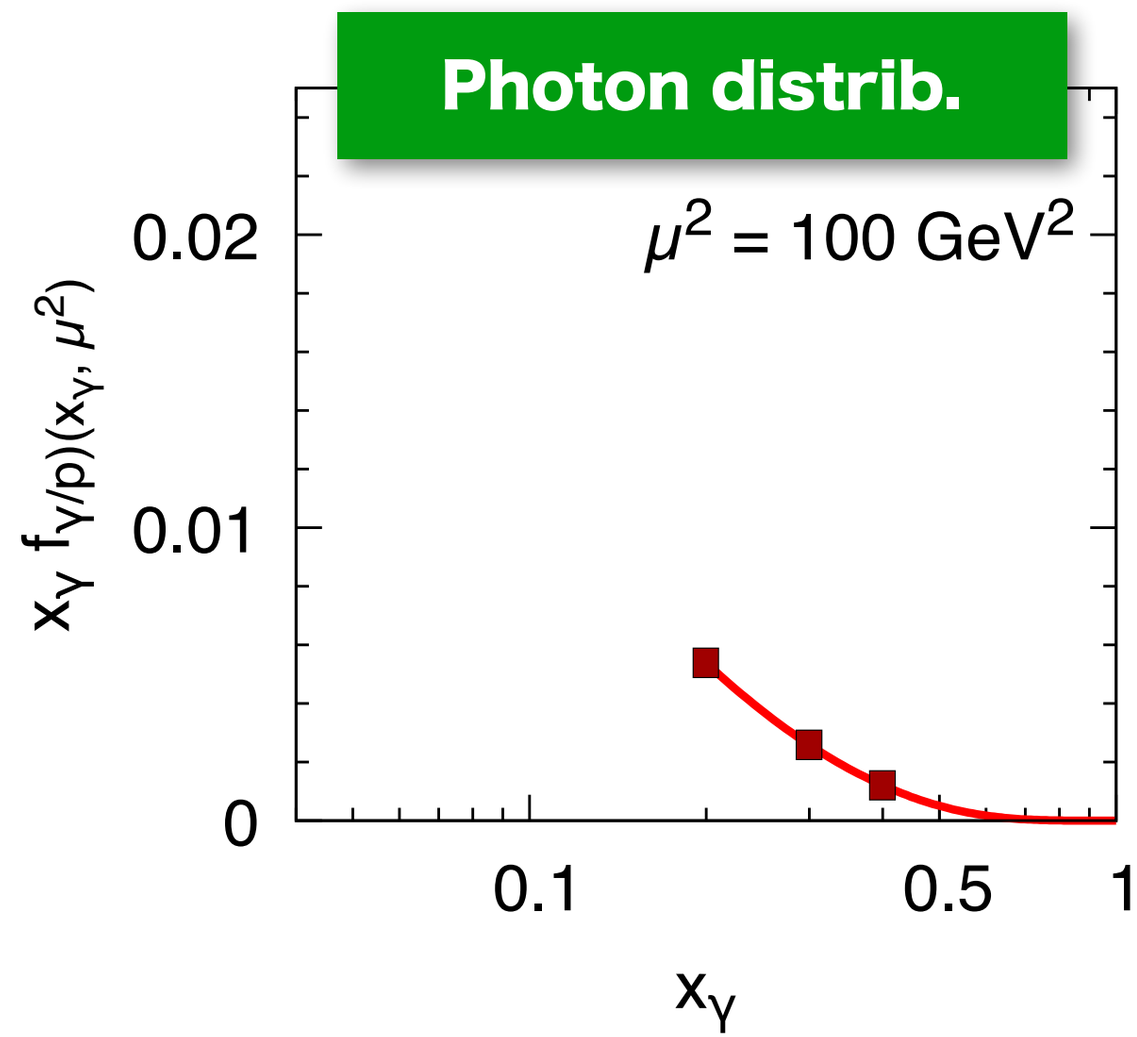






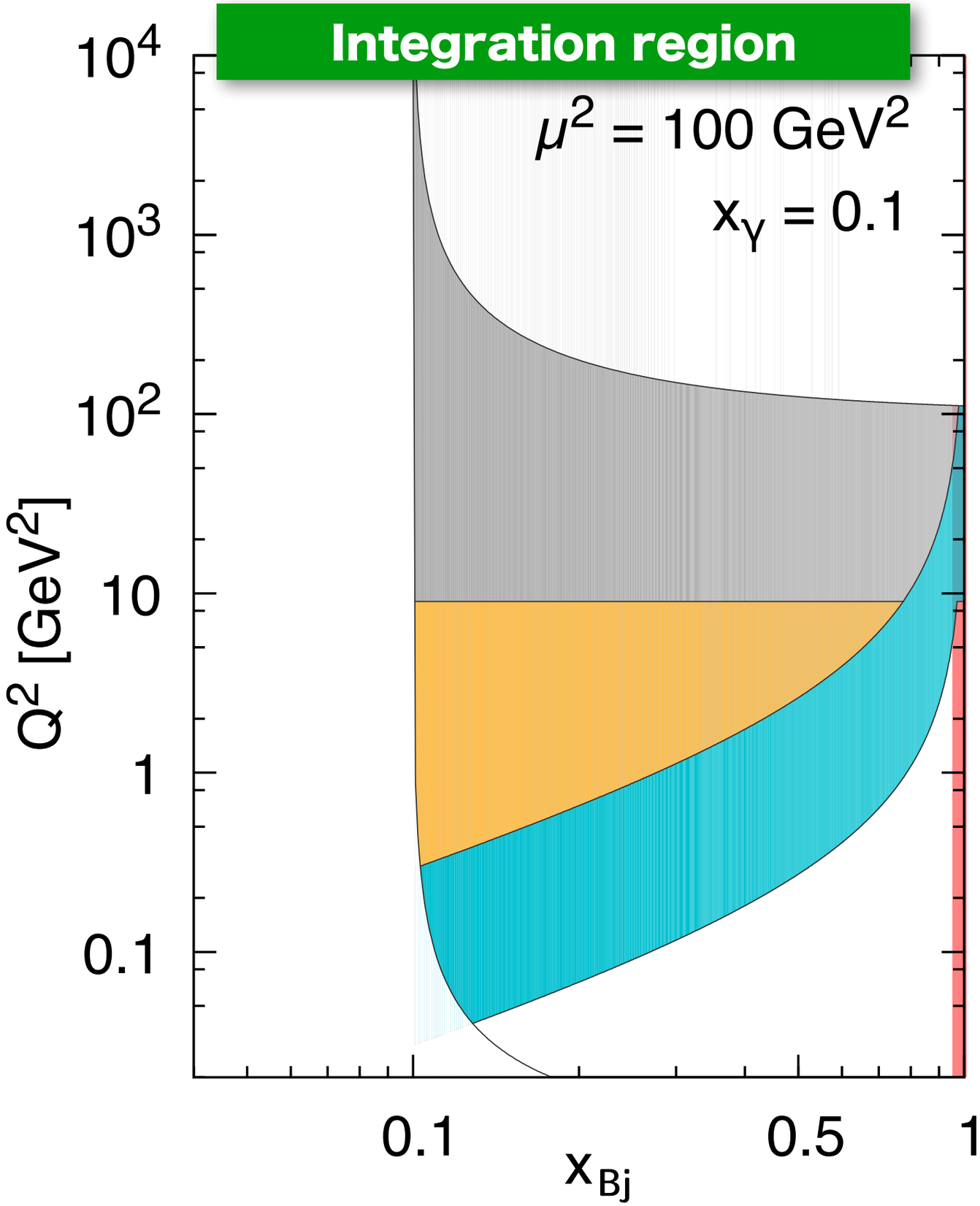
INTEGRATION REGION

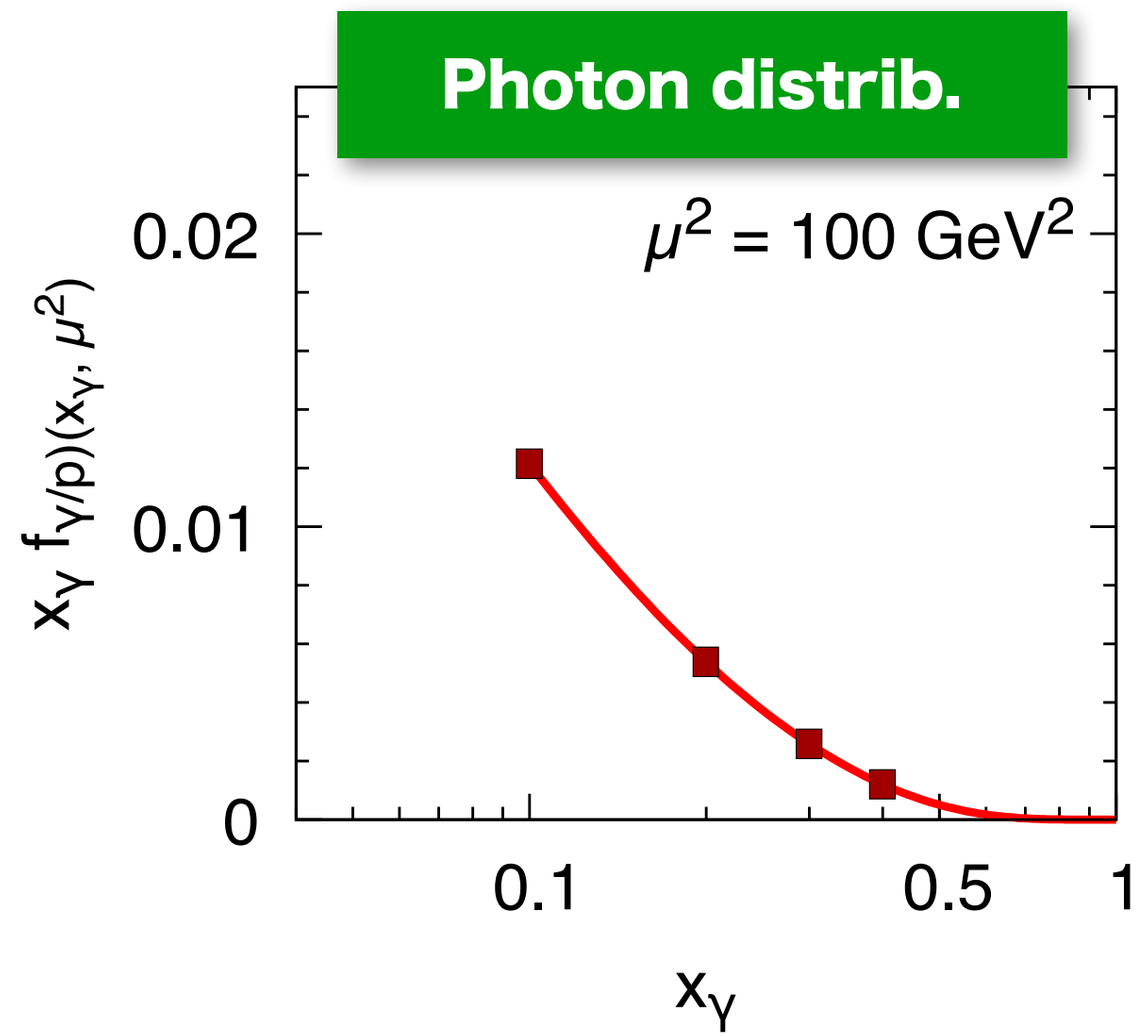
- depends on momentum fraction of the photon (x_γ) and factorisation scale (μ^2)

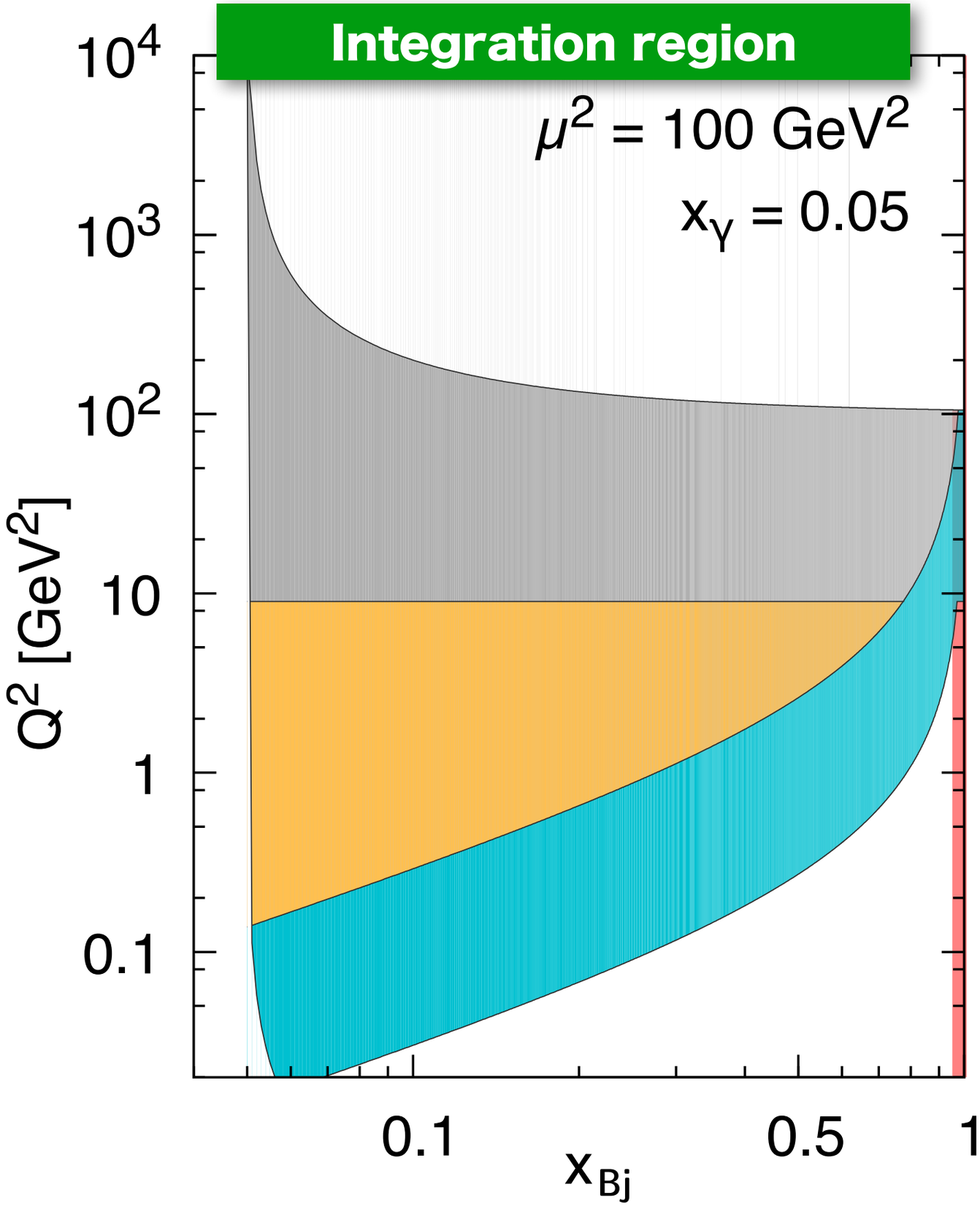


INTEGRATION REGION

- depends on momentum fraction of the photon (x_γ) and factorisation scale (μ^2)







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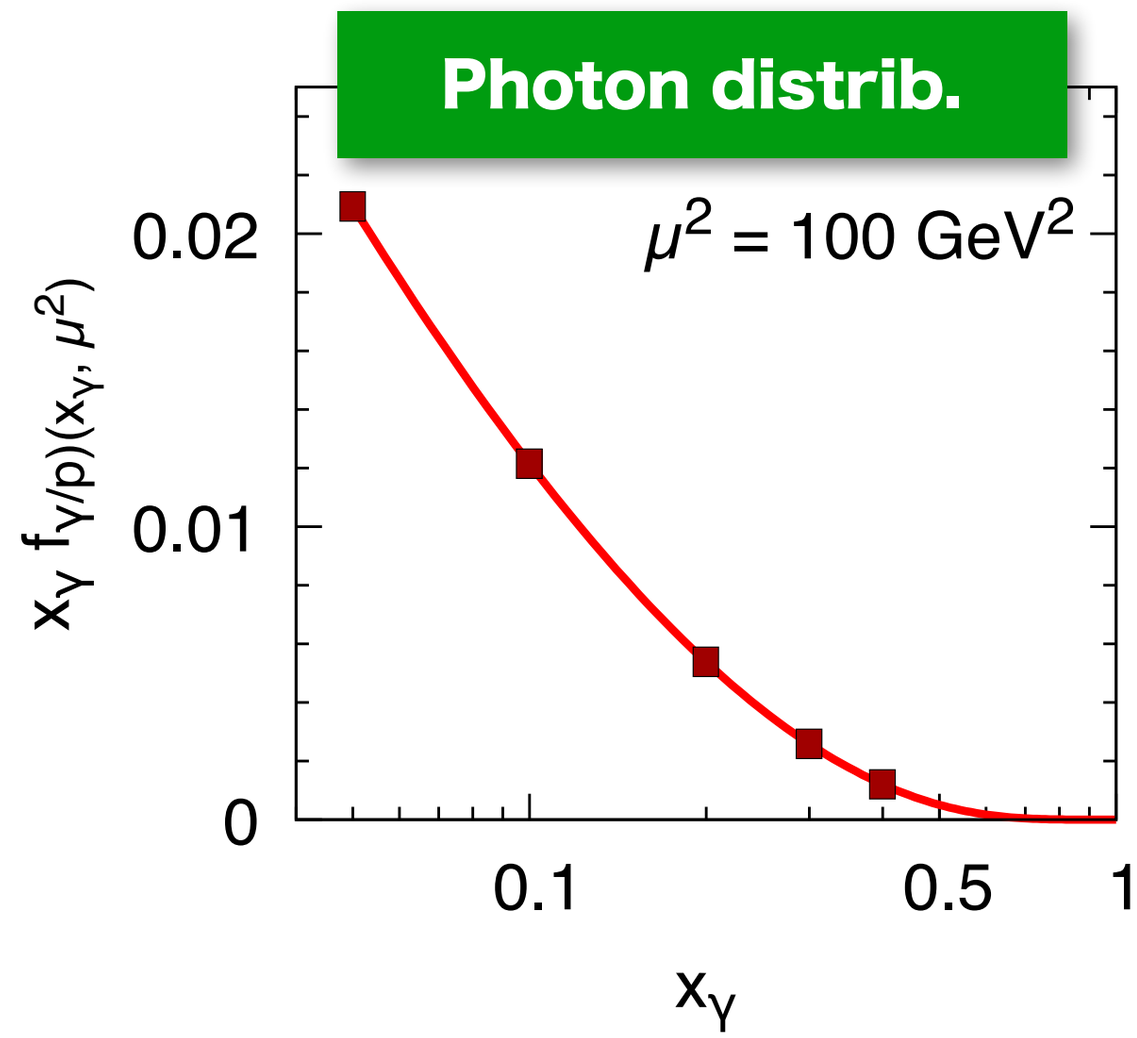
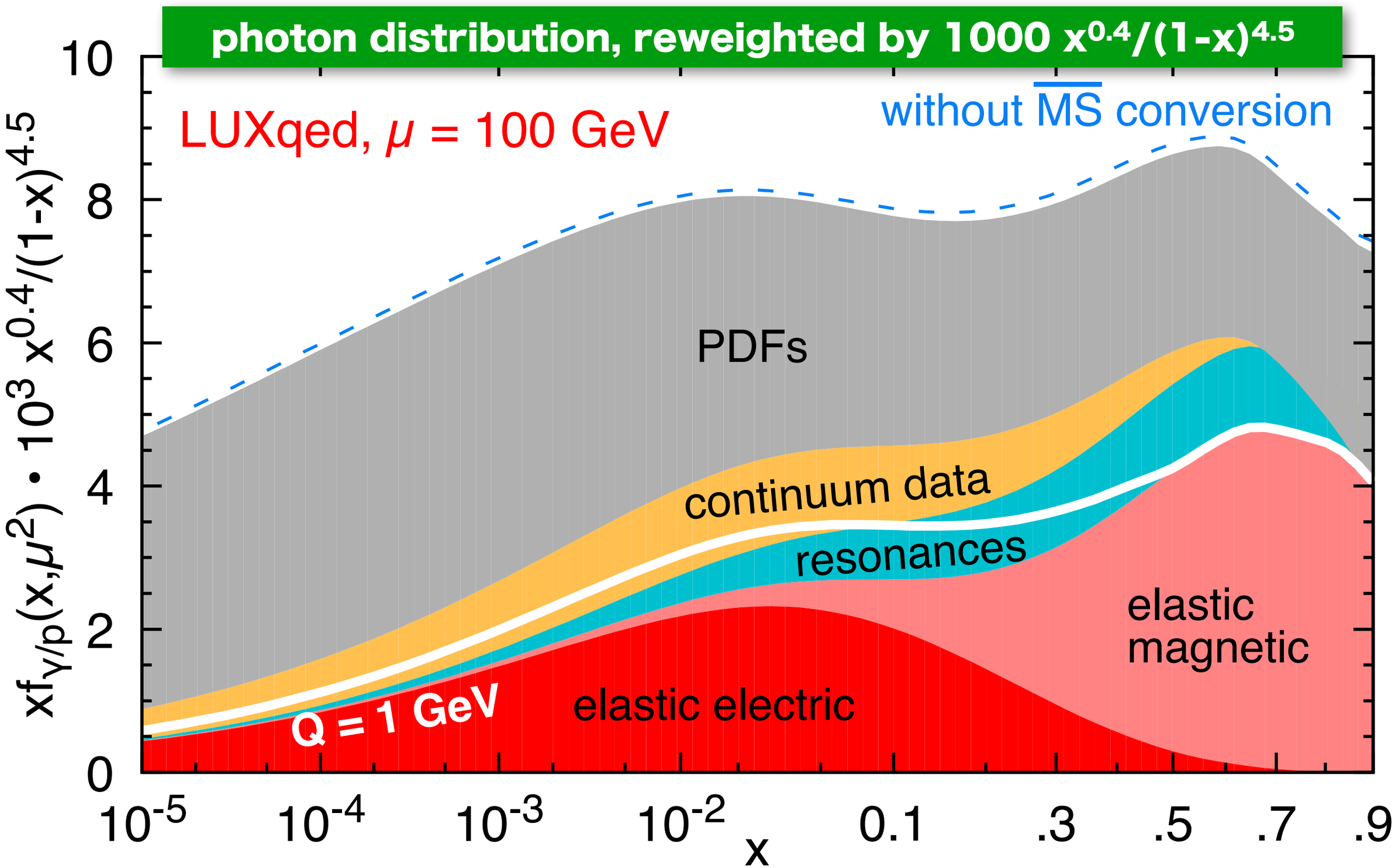
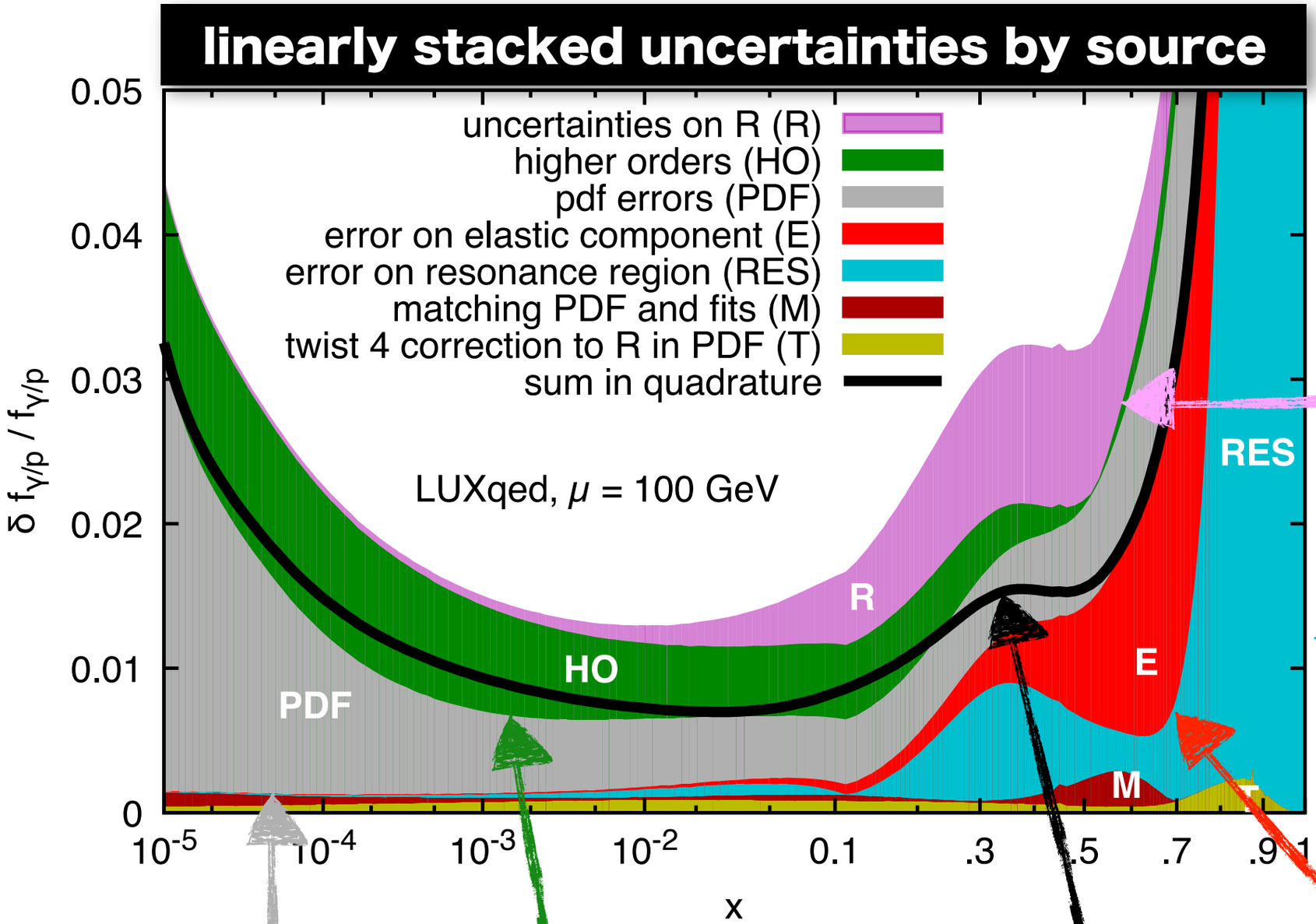


SEPARATE CONTRIBUTIONS TO PHOTON PDF



photon uncertainties (aim to be conservative & pragmatic)



±50% on R ($\sim F_L/(F_2\text{-}F_L)$) in low-$Q^2$ continuum and resonance regions

replace CLAS resonance fit with Christy-Bosted

standard PDF uncertainty

treatment of upper limit of Q^2 integral ($\mu^2/(1\text{-}z)$ v. μ^2)

final total uncertainty ~ 1 – 2%
(sum in quad. of all sources)

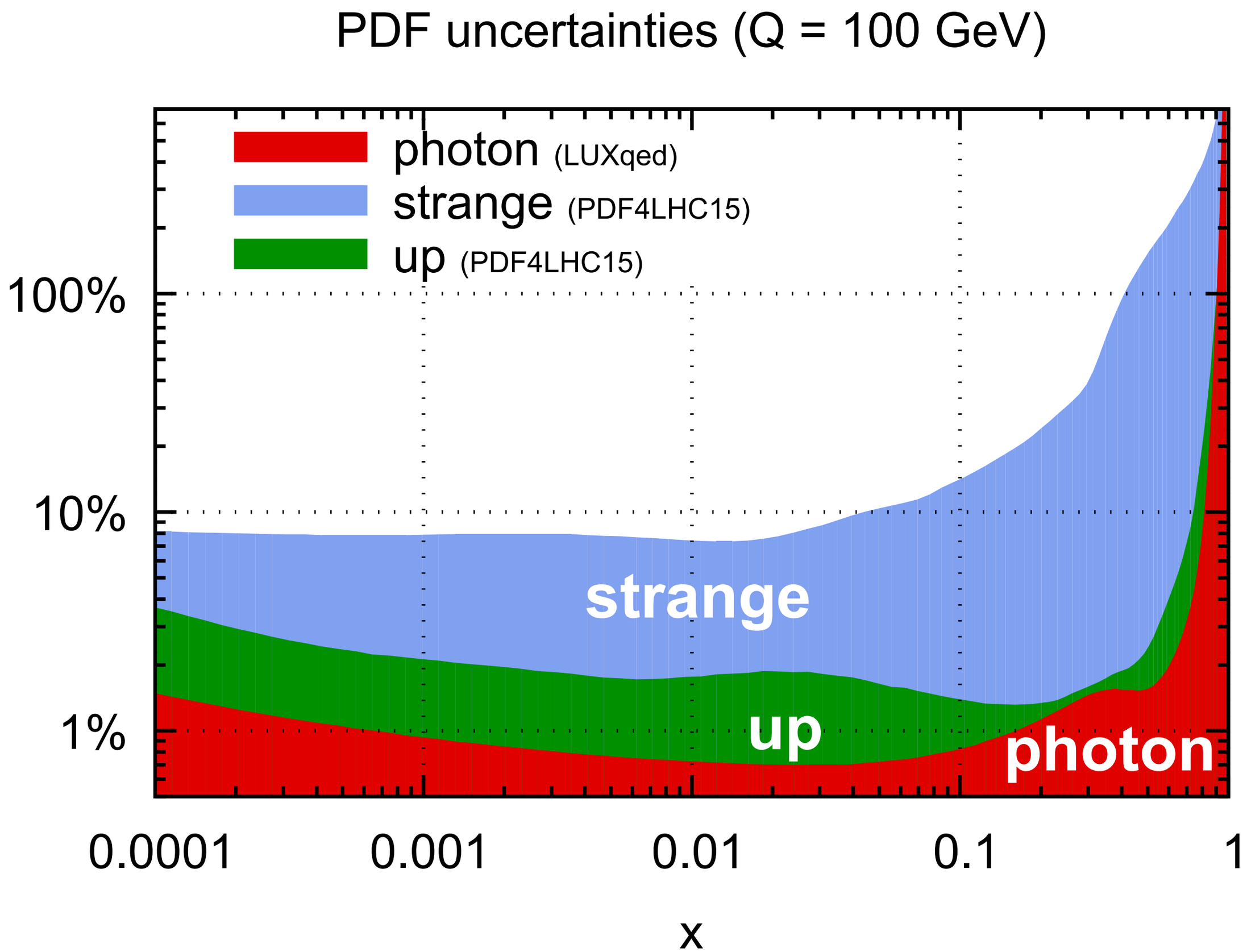
uncertainty on elastic component (quoted ⊕ unpol./pol.)

PHOTON PDF ESTIMATES (not exhaustive)

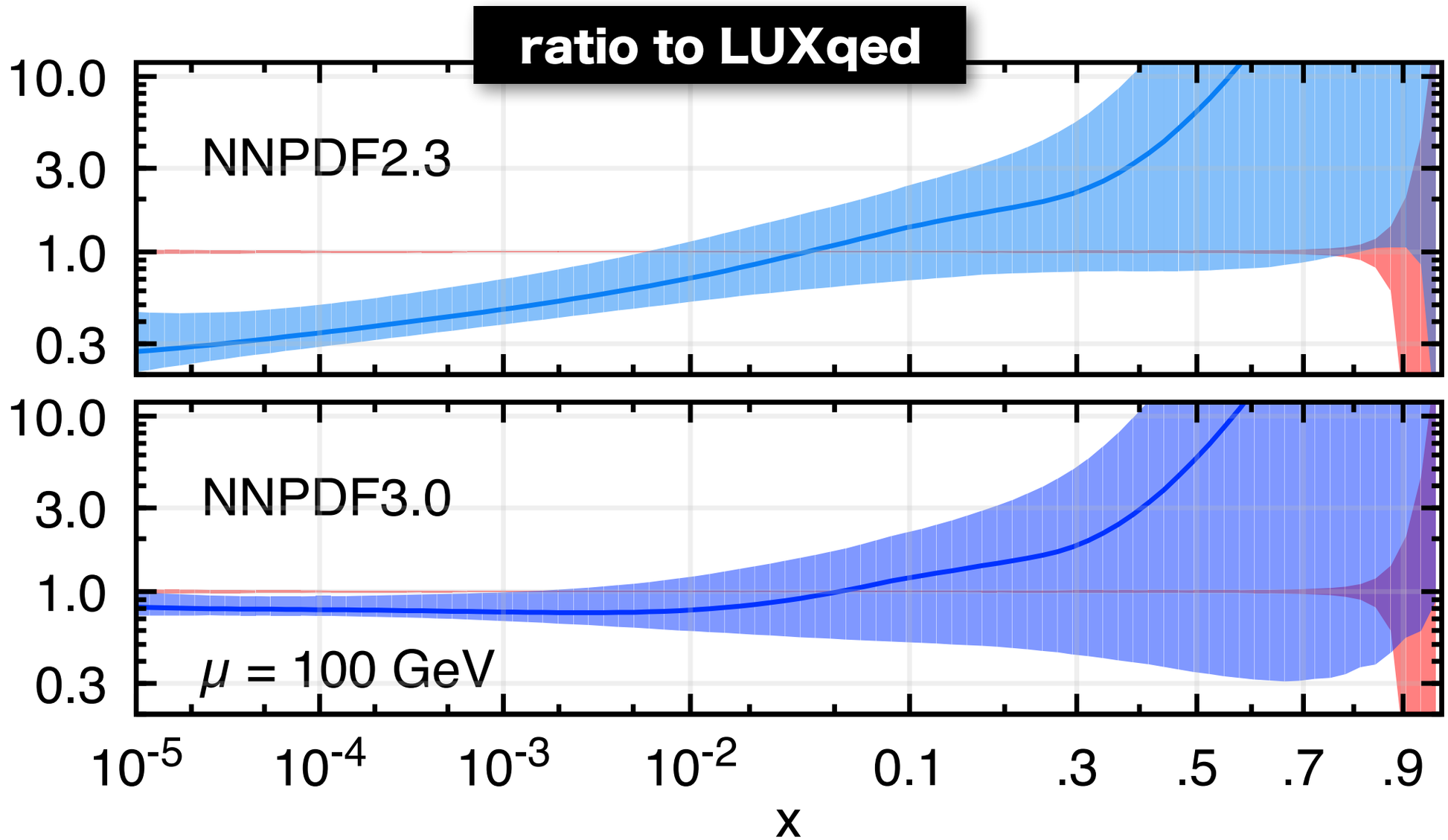
	elastic	inelastic	in LHAPDF?
Gluck Pisano Reya 2002	dipole	model	✘
MRST2004qed	✘	model	✓
NNPDF23qed	no separation; fit to data		✓
CT14qed	✘	model (data-constrained)	✓
CT14qed_inc	dipole	model (data-constrained)	✓
Martin Ryskin 2014	dipole (only electric part)	model	✘
Harland-Lang, Khoze Ryskin 2016	dipole	model	✘
LUXqed 2016	data	data	✓

examine result

PHOTON UNCERTAINTY (1-2%) COMPARED TO OTHER FLAVOURS



other PDFs v. LUXqed



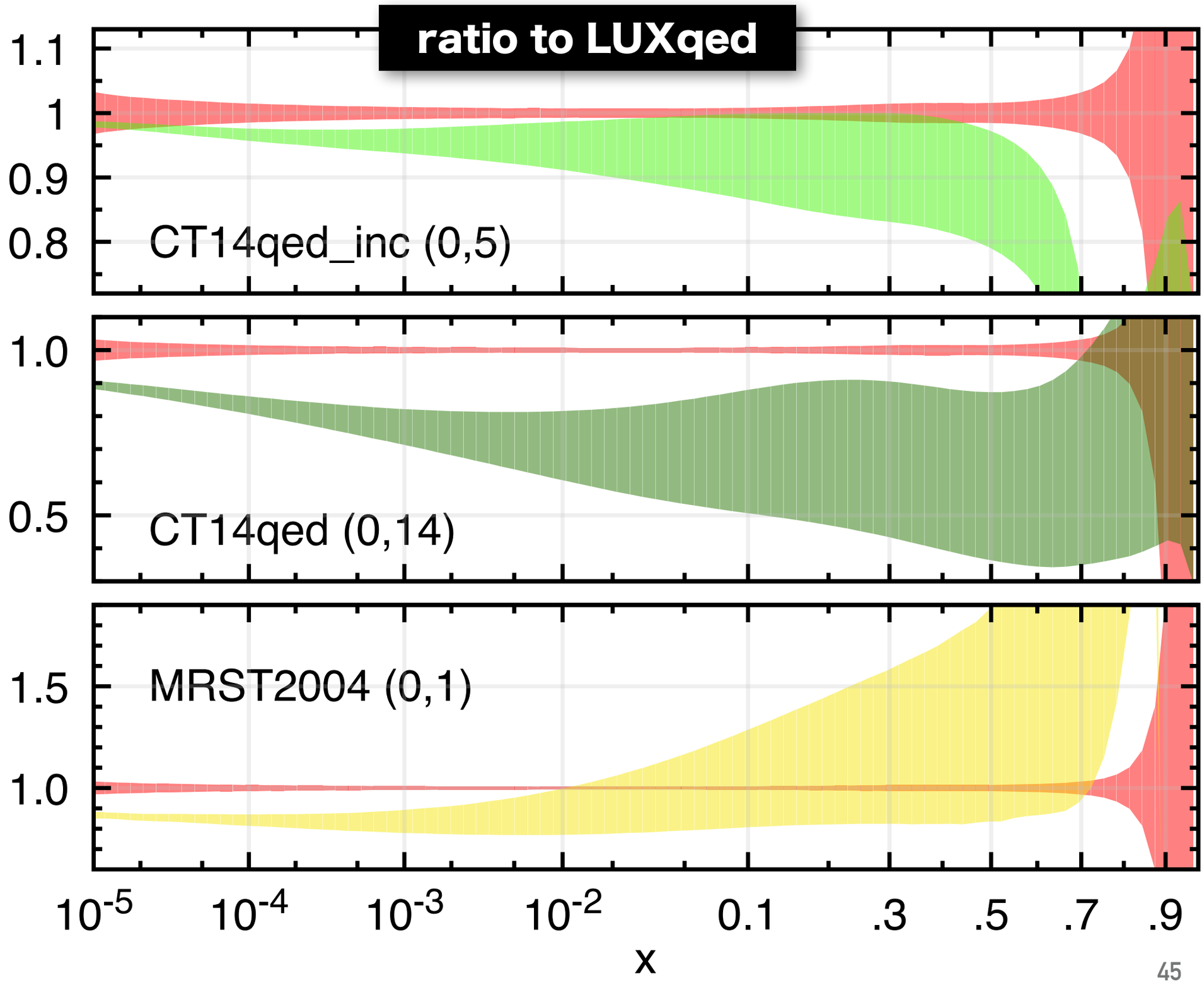
central NNPDF result much higher at large x (but consistent within errors)

at small x, with corrected evolution (NNPDF30), about 20% smaller

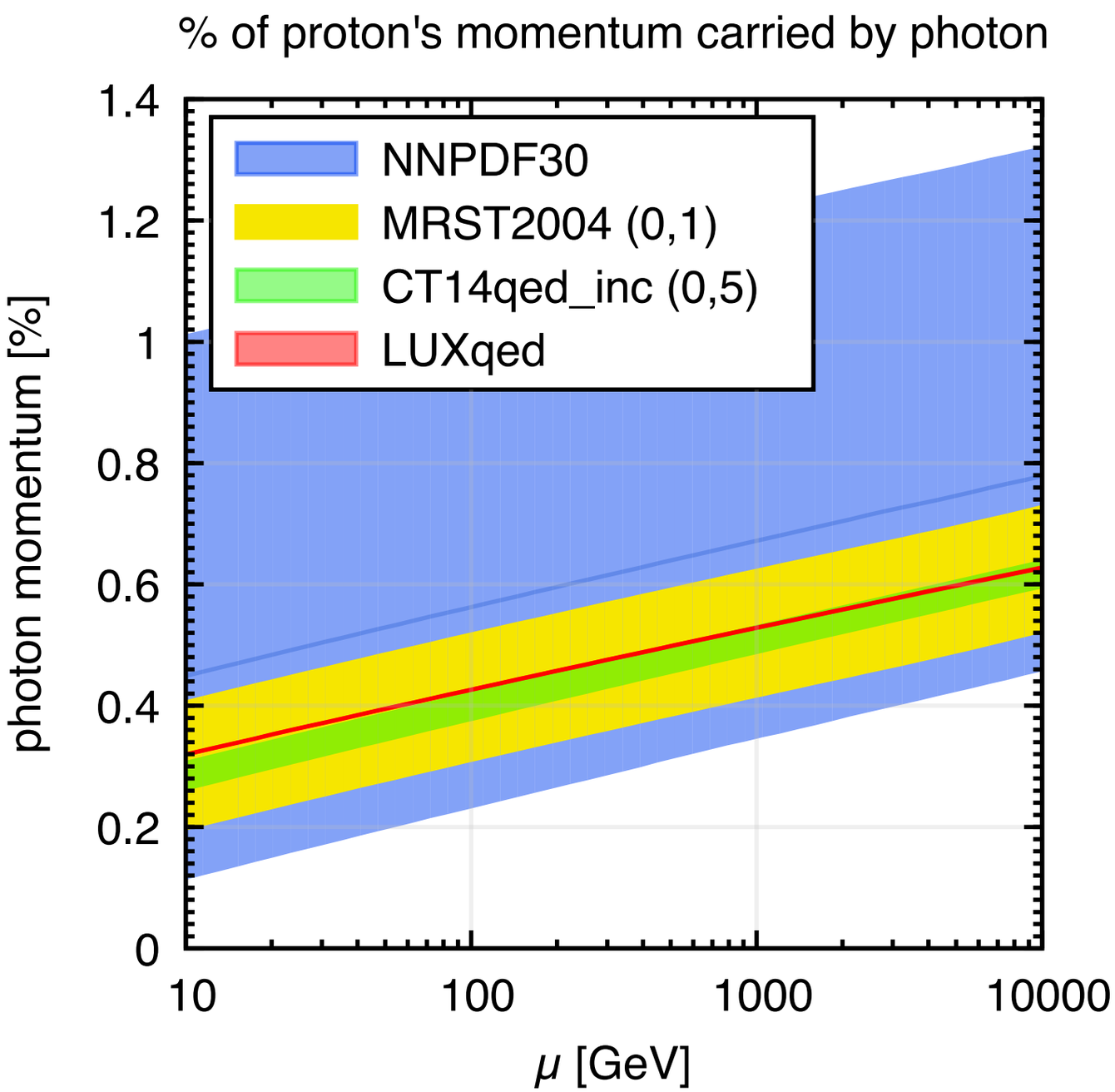
other PDFs v. LUXqed

Others are numerically closer

Error bands don't always overlap with LUXqed, but within ~10-20%



MOMENTUM CARRIED BY PHOTON



momentum (μ = 100 GeV)	
gluon	46.8 ± 0.4%
up valence	18.2 ± 0.3%
down valence	7.5 ± 0.2%
light sea quarks	20.7 ± 0.4%
charm	4.0 ± 0.1%
bottom	2.5 ± 0.1%
photon	0.426 ± 0.003%

`LUXqed_plus_PDF4LHC15_nnlo_100`

(1+107 members, symmhessian, errors handled by LHAPDF out of the box)

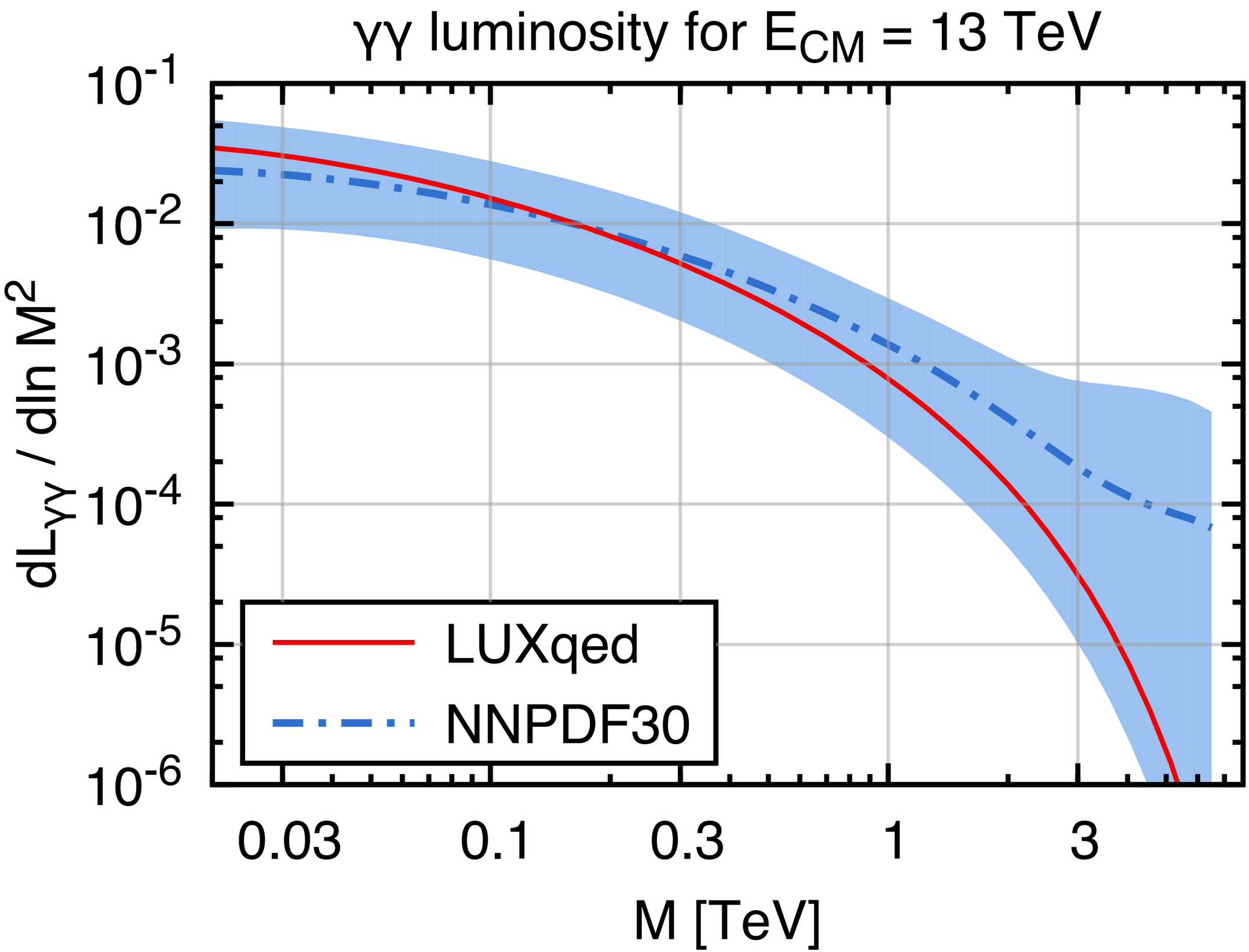
applications

APPLICATION TO HIGGS PHYSICS

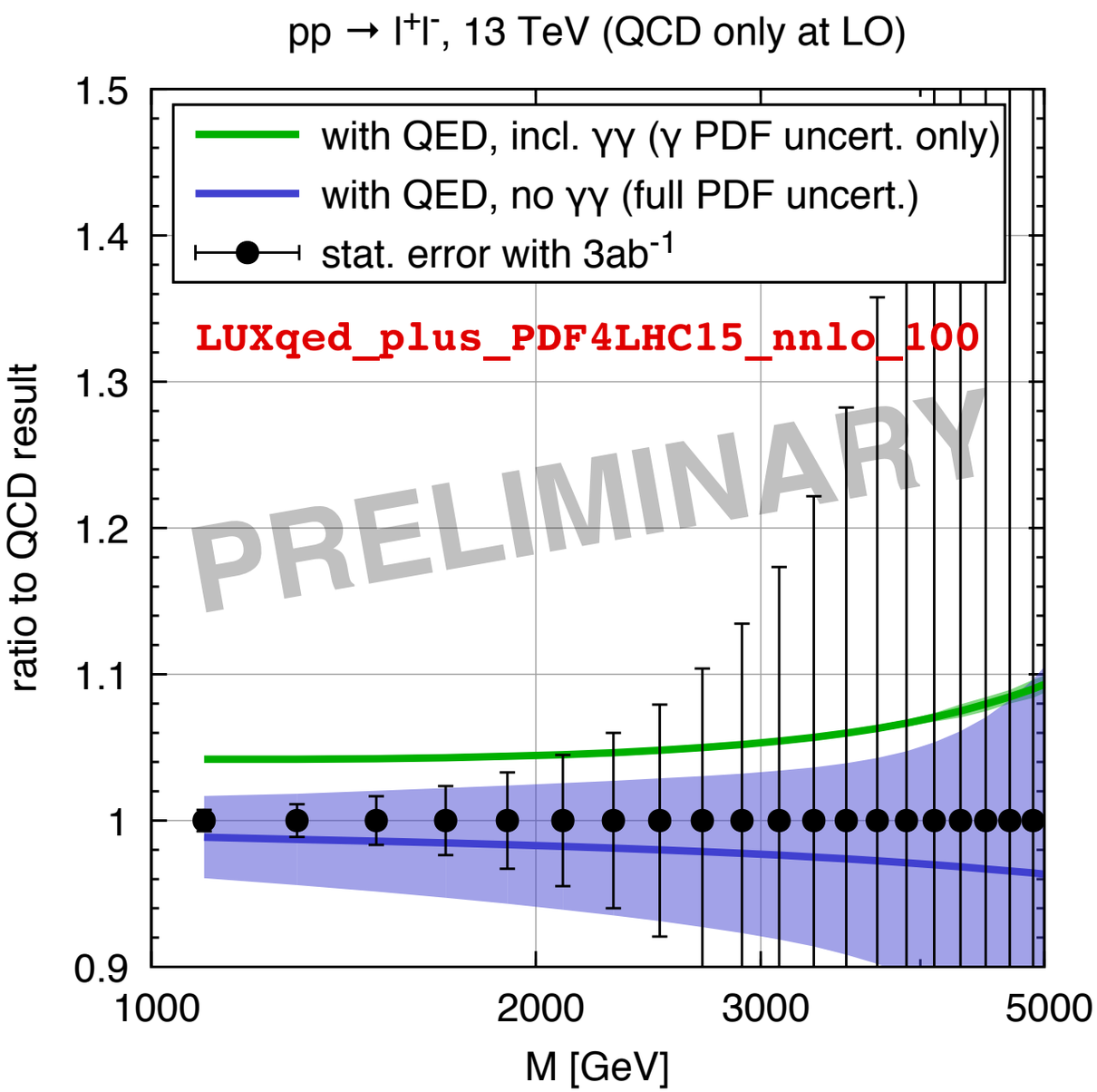
$pp \to H\, W^+ (\to l^+\nu) + X$ at 13 TeV	
non-photon induced contributions	91.2 ± 1.8 fb
photon-induced contribs (NNPDF23)	$6.0\ ^{+4.4}_{-2.9}$ fb
photon-induced contribs (LUXqed)	4.4 ± 0.1 fb

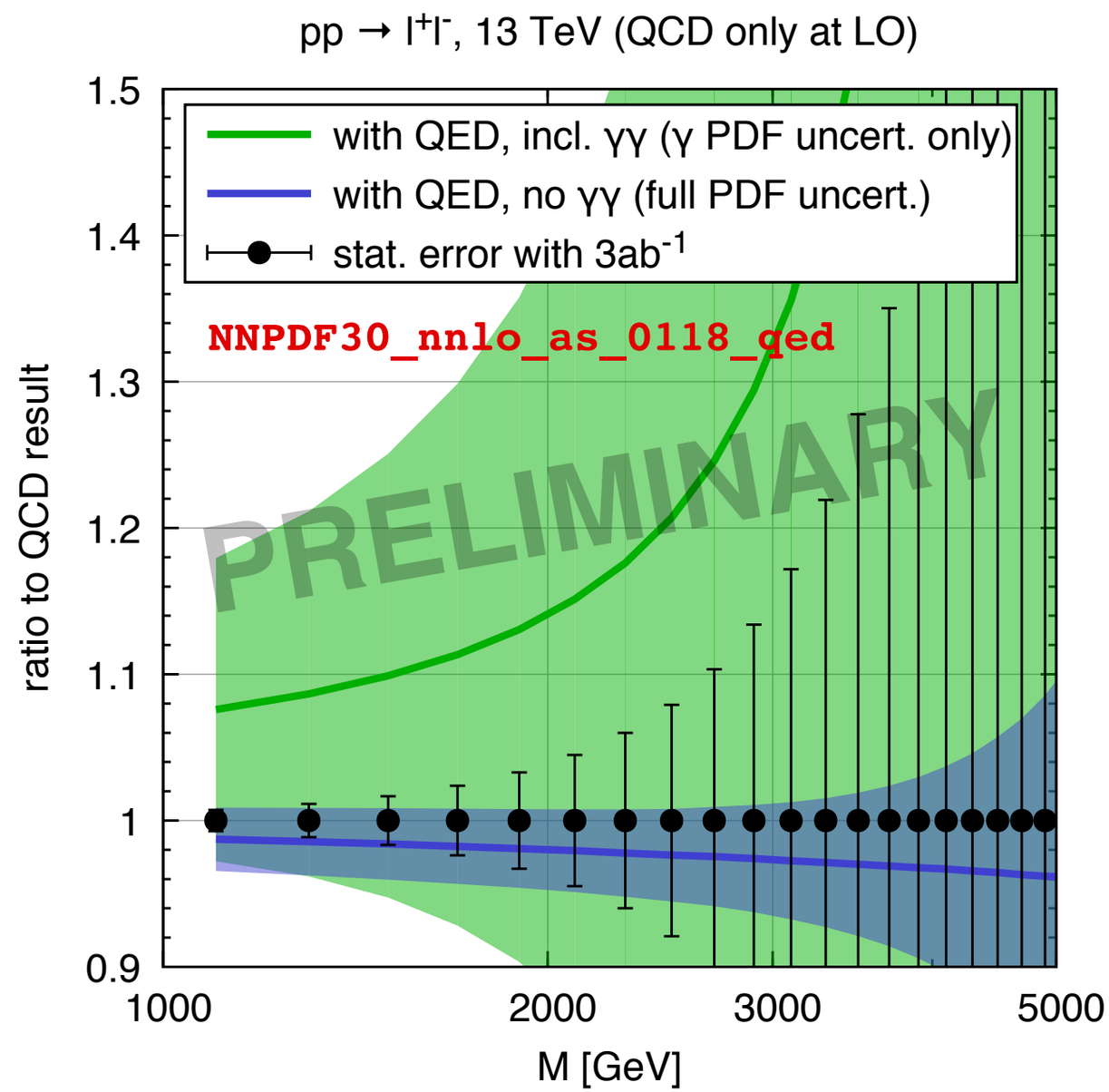
non-photon numbers from LHCHXSWG (YR4)
including PDF uncertainties

γγ luminosity



di-lepton spectrum





LUXQED photon has few % effect on di-lepton spectrum and negligible uncertainties

conclusions & resources

RESOURCES

- `LUXqed_plus_PDF4LHC15_nnlo_100` set available from LHAPDF
- Additional plots and validation info available from `http://cern.ch/luxqed`
- Preliminary version of HOPPET DGLAP evolution code with QED (order α and $\alpha\alpha_s$) corrections available from hepforge:

```
svn checkout http://hoppet.hepforge.org/svn/branches/qed hoppet-qed
```

(look at `tests/with-lhapdf/test_qed_evol_lhapdf.f90` for an example; interface may change, documentation missing)

CLOSING REMARKS

- distribution of photons in the proton depends on the **non-perturbative QCD** physics of the proton
- But **perturbative QED** enables you to deduce the photon density from measured (non-pert.) proton structure functions

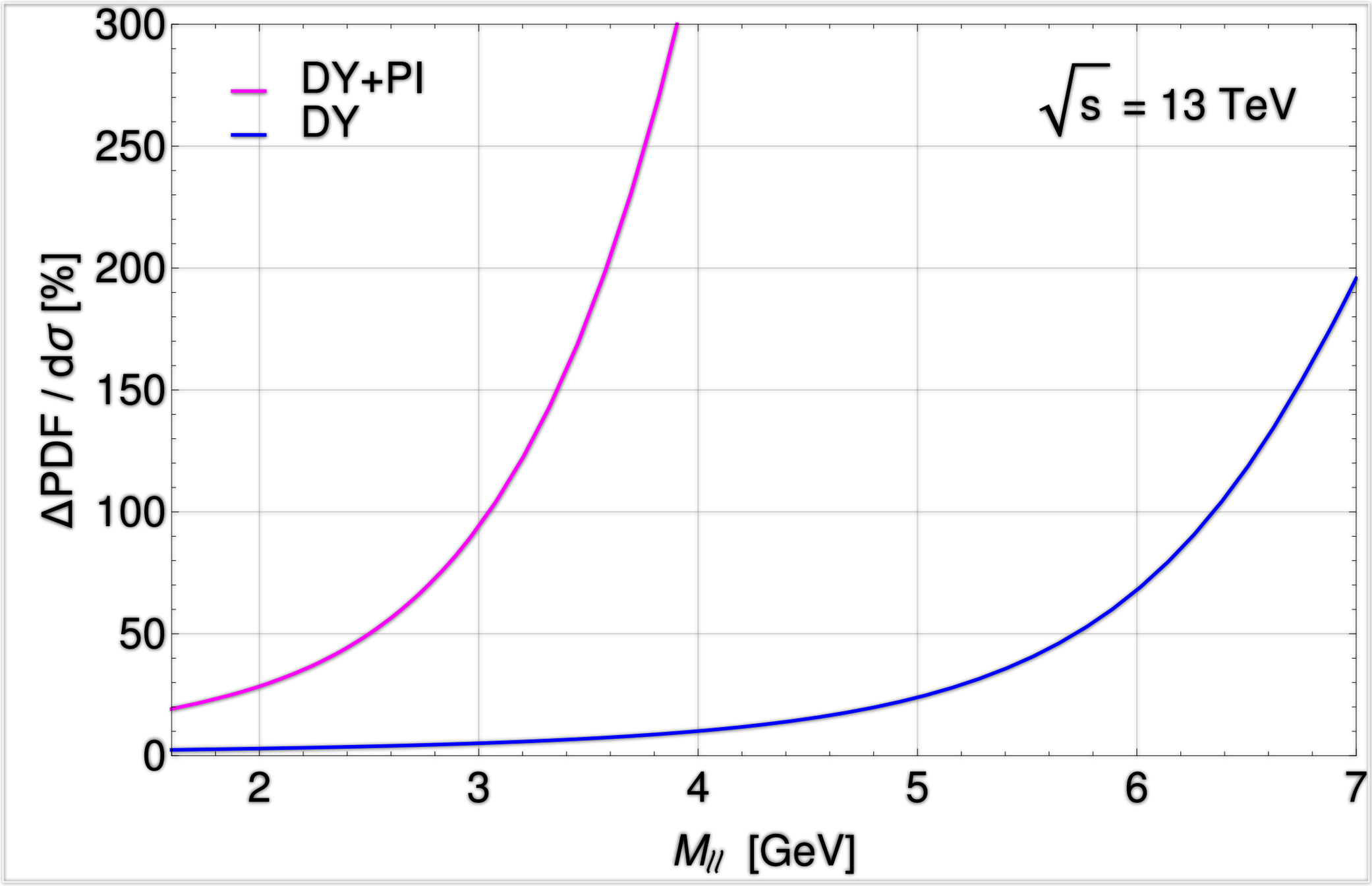
"If you think about it, it's awesome: we are made of protons, and protons are, in some part, made of light... And now we know how much of it."

blog post by Tommaso Dorigo

extra slides

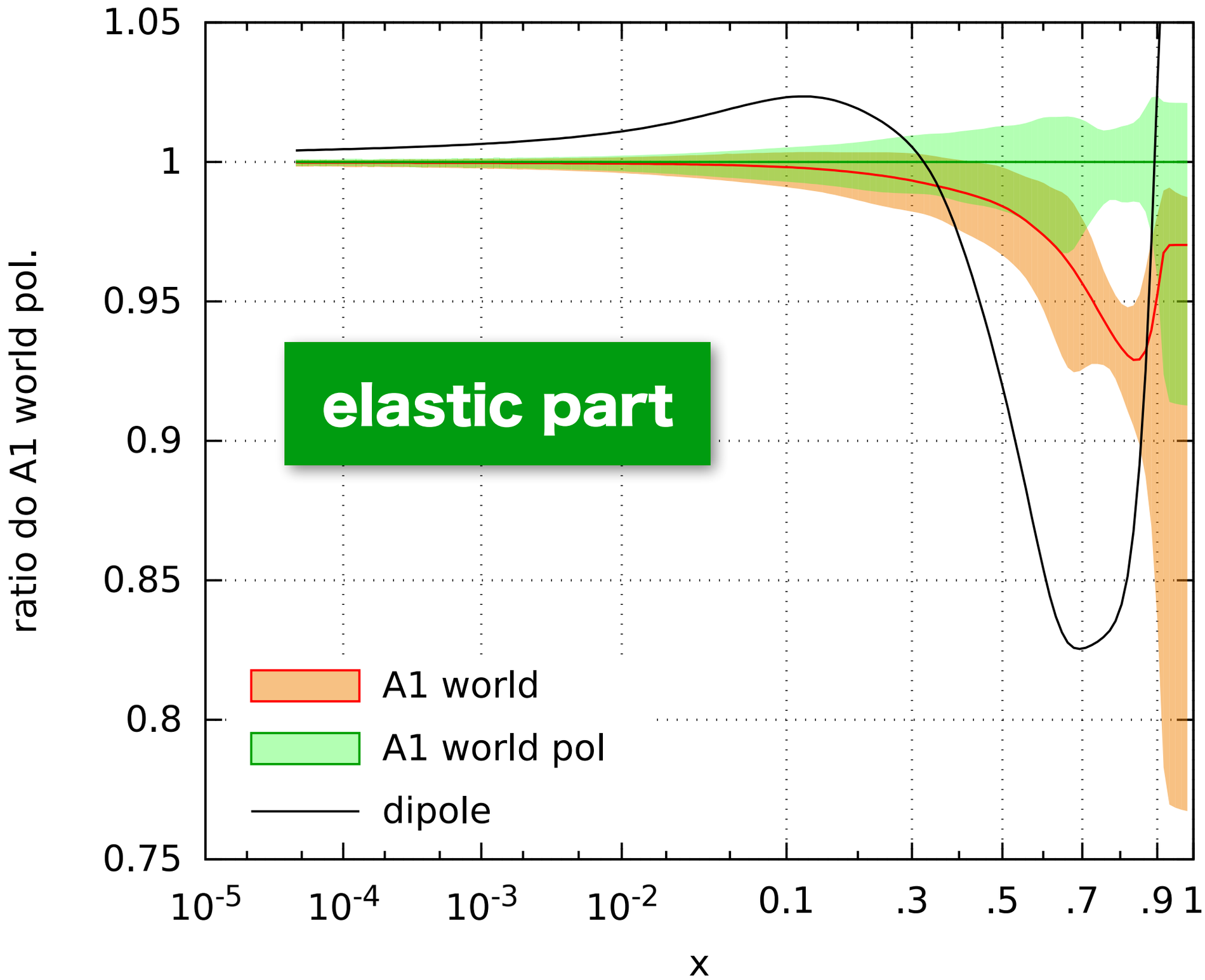
1606.06646v1

Elena Accomando,[1,2,*] Juri Fiaschi,[1,2,†] Francesco Hautmann,[2,3,‡]
Stefano Moretti,[1,2,§] and C.H. Shepherd-Themistocleous[1,2,¶]

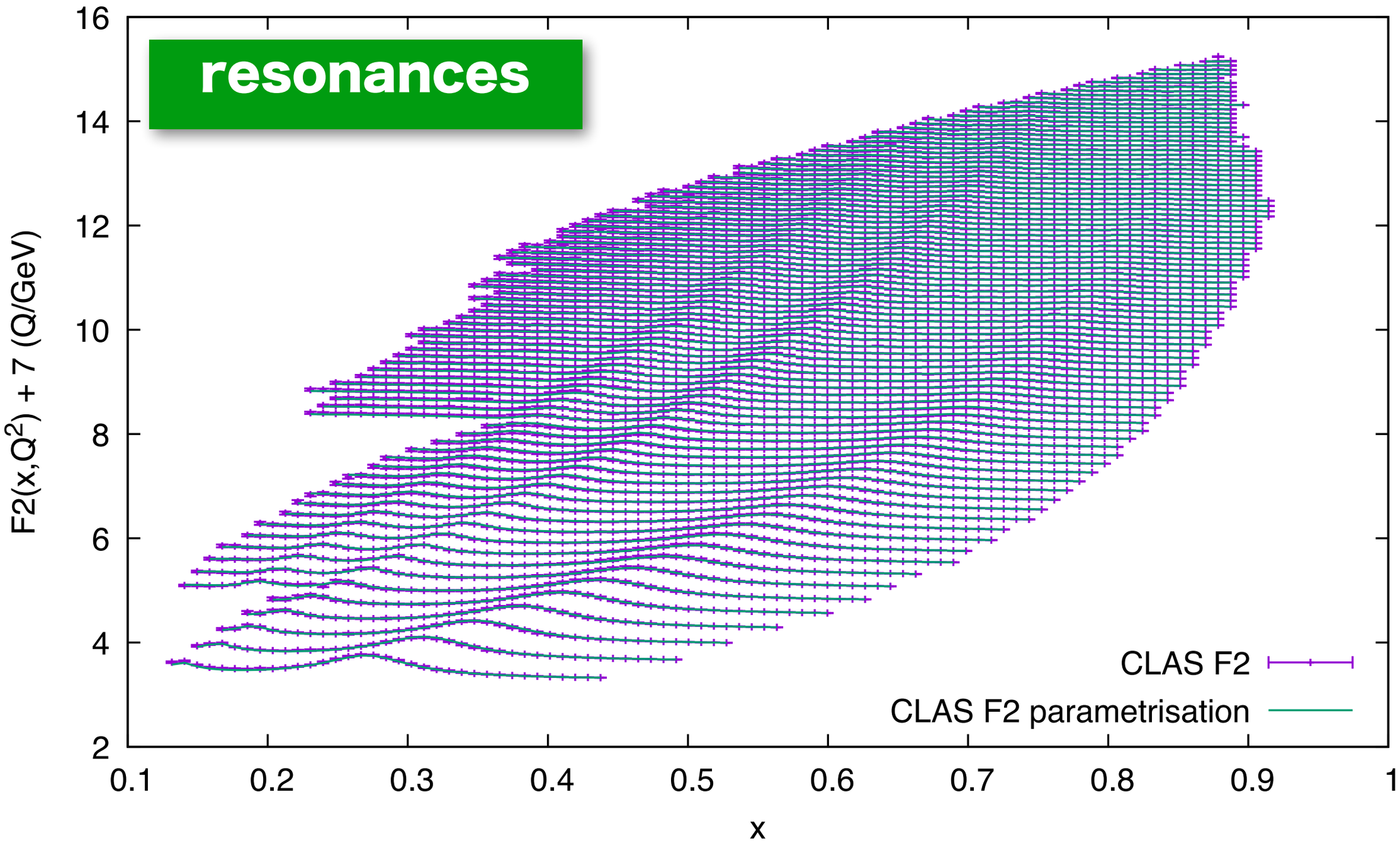


input data & procedures

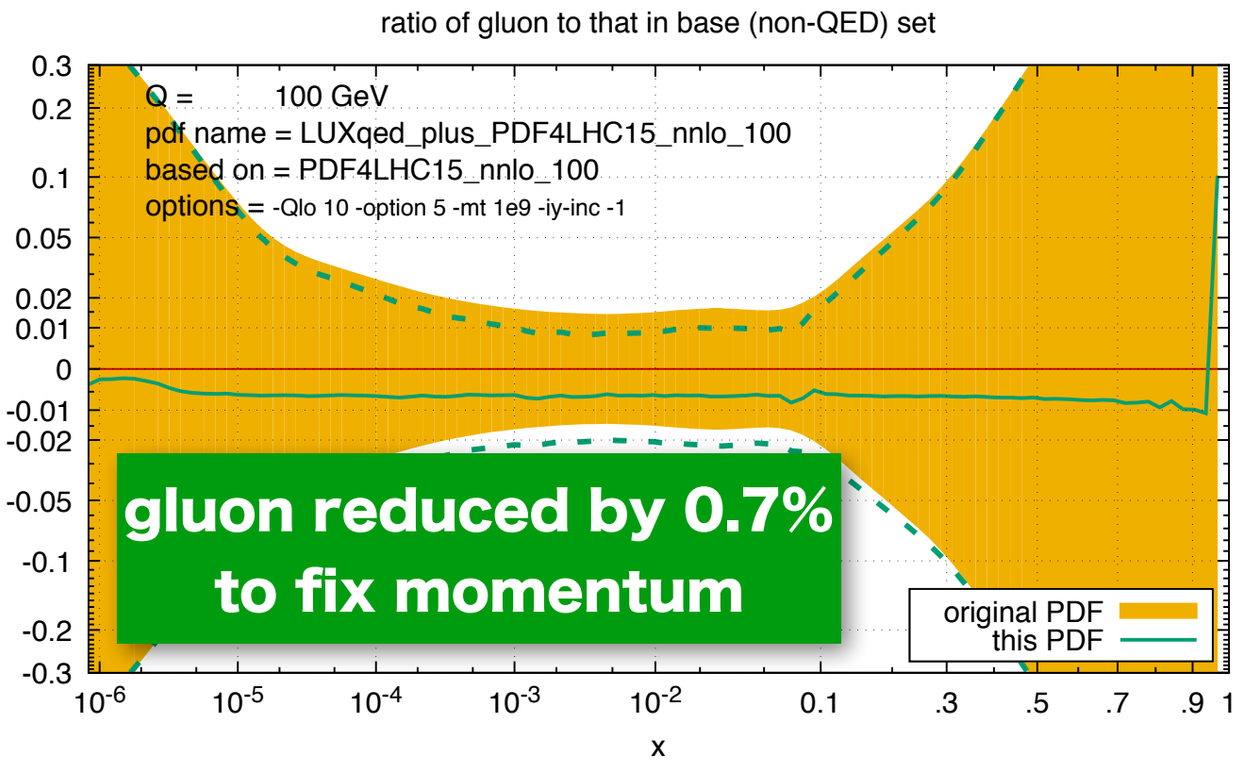
ELASTIC COMPONENT & COMPARISON TO "DIPOLE" MODEL

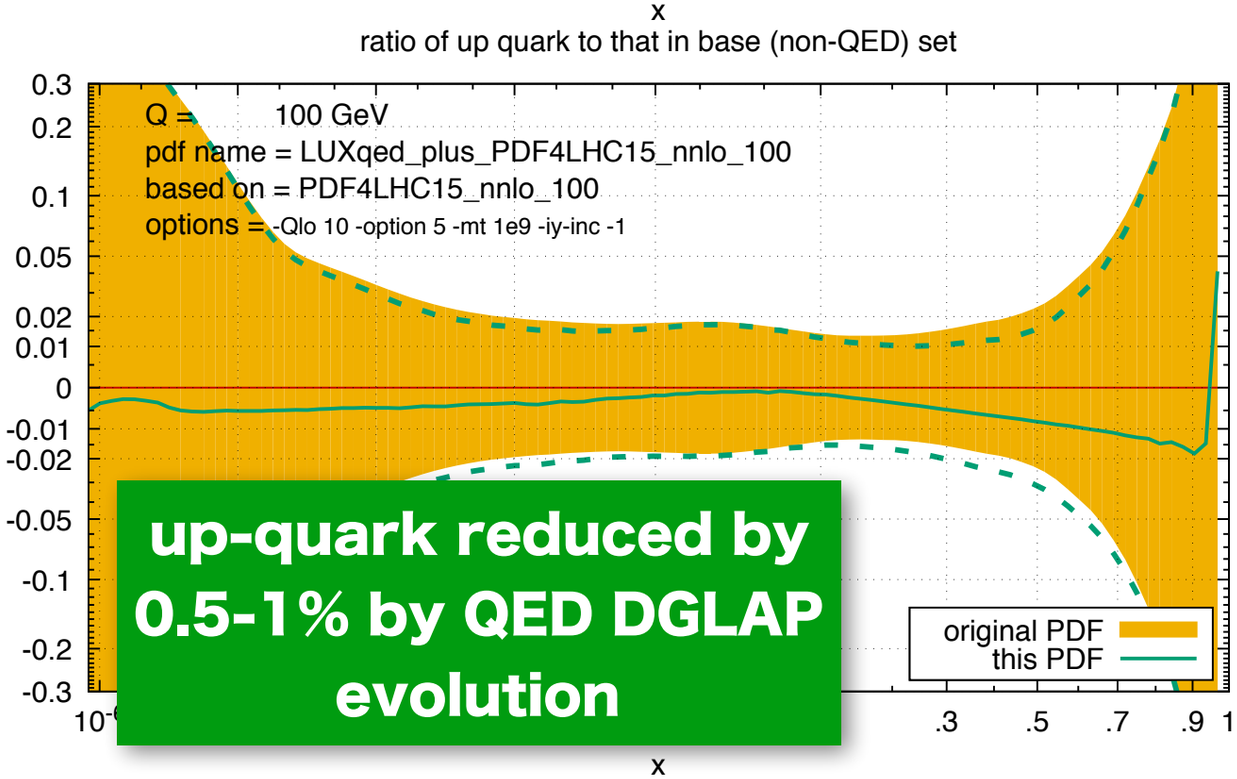


CLAS DATA


resonances
16
14
12
10
8
6
4
2
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x
F2(x,Q2) + 7 (Q/GeV)
CLAS F2
CLAS F2 parametrisation

MATCHING PROCEDURE FOR FULL SET OF PARTONS



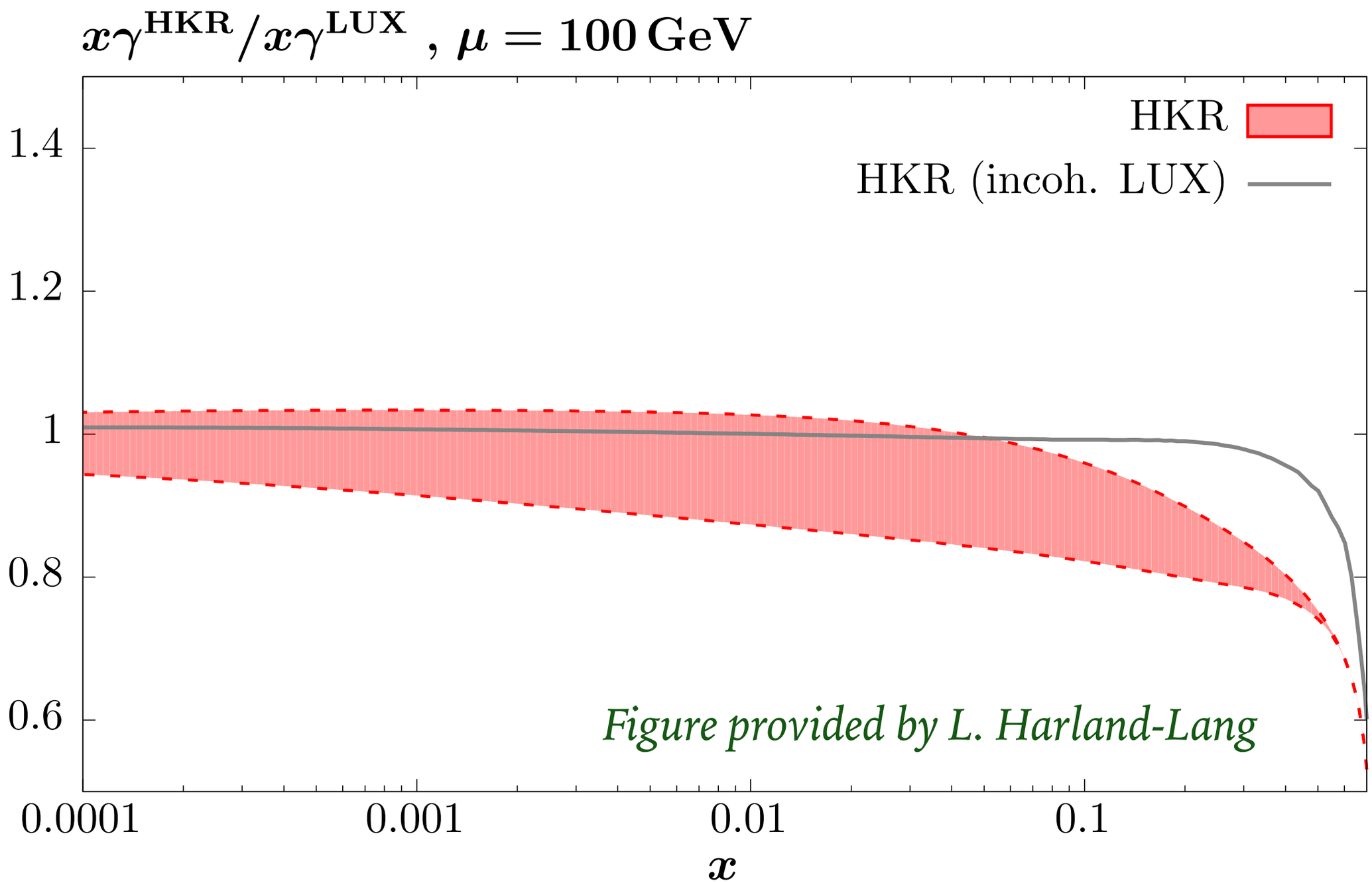


- evaluate master eqn. for $\mu = 100$ GeV (with default `PDF4LHC15_nnlo` partons)
- Do $O(\alpha\alpha_s)$ photon evolution down to $\mu = 10$ GeV (other partons: pure QCD evln.)
- Adjust momentum sum-rule by rescaling gluon $g(x) \to 0.993 g(x)$
- Evolve back up with NNLO-QCD & $O(\alpha\alpha_s)$ QED for all partons

better approach would be full PDF re-fit for QCD partons incl. EW/QED corrections & LUXqed photon

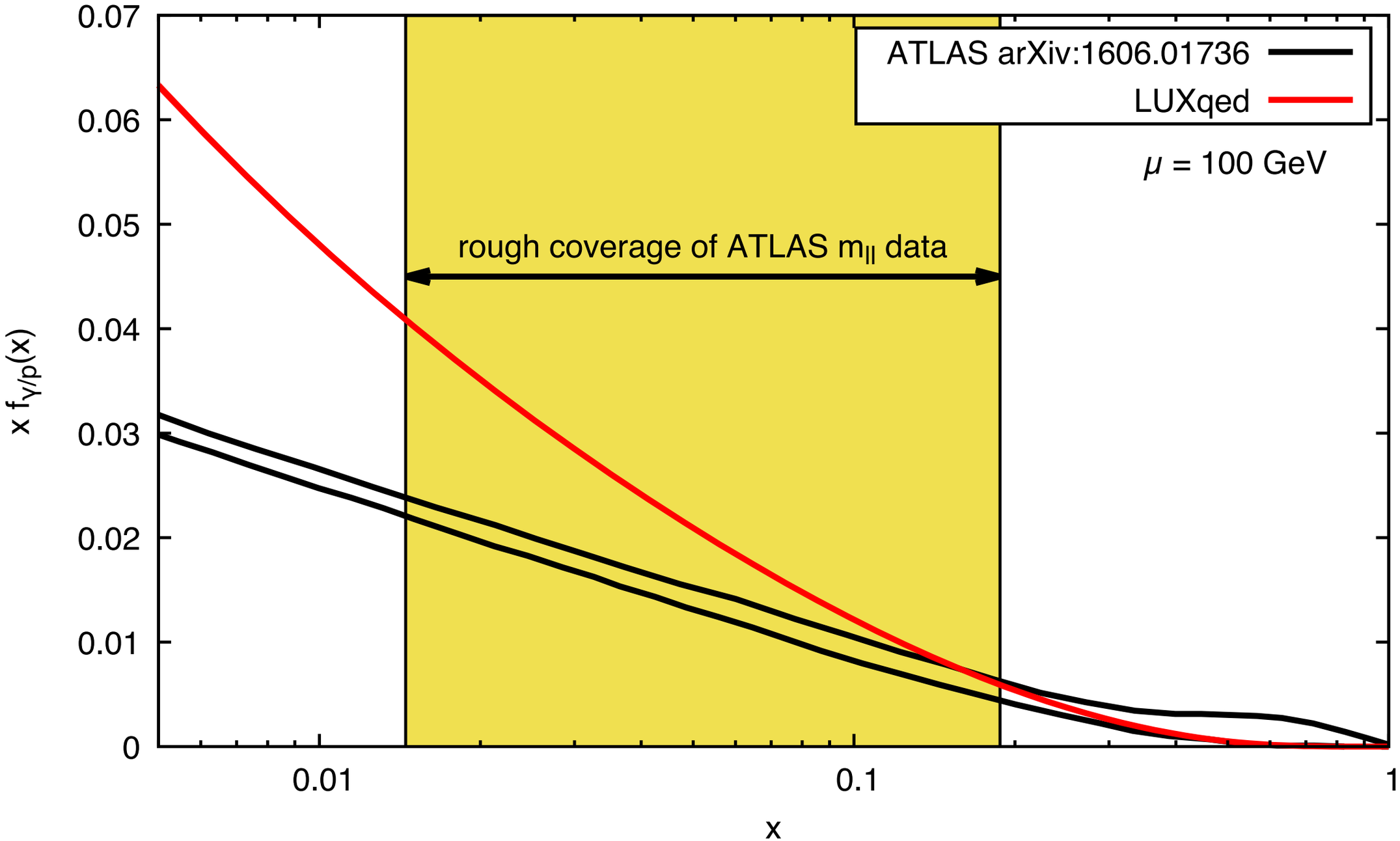
comparisons to others

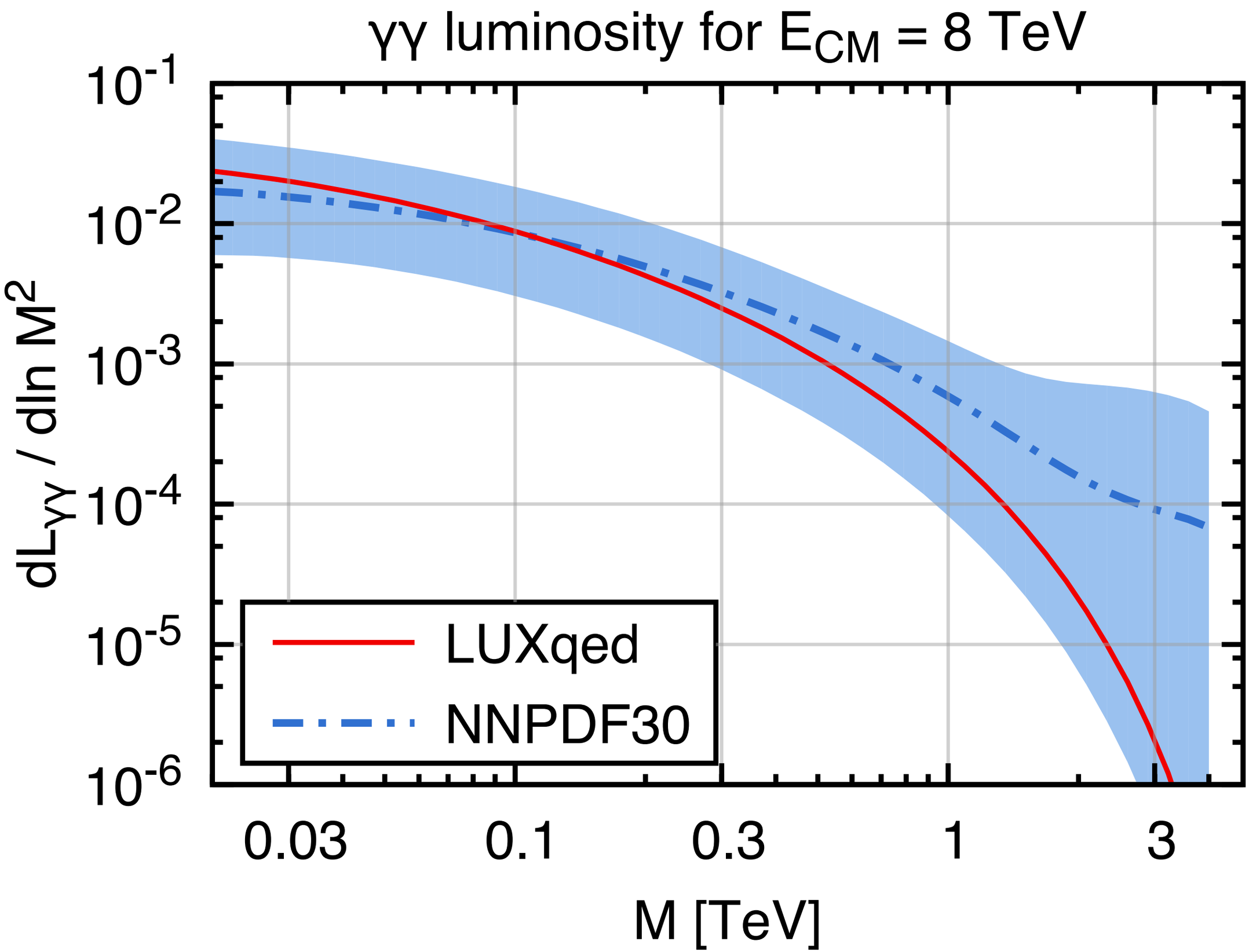
ratio of HKR (1607.04635) to LUXqed



HKR based on elastic contribution (dipole approx) + model for inelastic part + evolution

ratio of ATLAS photon (1606.01736) to LUXqed



ATLAS result based on reweighting of NNPDF23 with high-mass (M_{ll} > 116 GeV) data

γγ luminosity