



**Universität  
Zürich<sup>UZH</sup>**

# **Vacuum stability in the Standard Model: Towards a four-loop precision analysis**

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# Outline

I. Introduction: Vacuum stability and the evolution of couplings

II. SM  $\beta$ -functions at 4 loops

1. Conceptual challenge:  $\gamma_5$ -treatment

2. Leading 4 loop contributions to  $\beta_{g_s}$  [JHEP 1602 (2016) 095]  
and leading 4 loop contributions to  $\beta_{y_t}$  and  $\beta_\lambda$  [JHEP 06 (2016) 175]

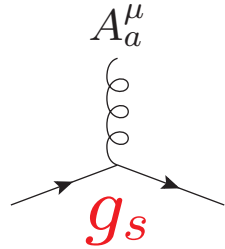
3. 4 loop calculations beyond the SM [arXiv:1608.08982]

III. Current status of the the vacuum stability question in the SM

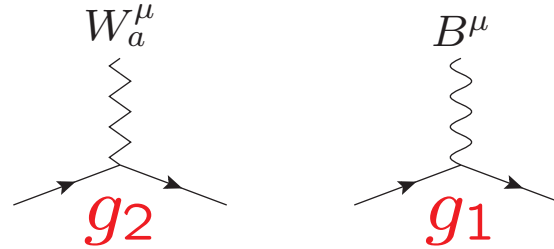
# I. Vacuum stability and the evolution of couplings

## Standard Model interactions:

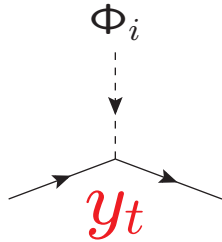
- strong:



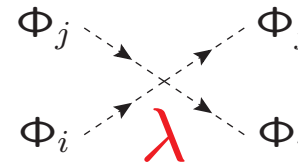
- electroweak:



- Yukawa:



- Higgs self-interaction:



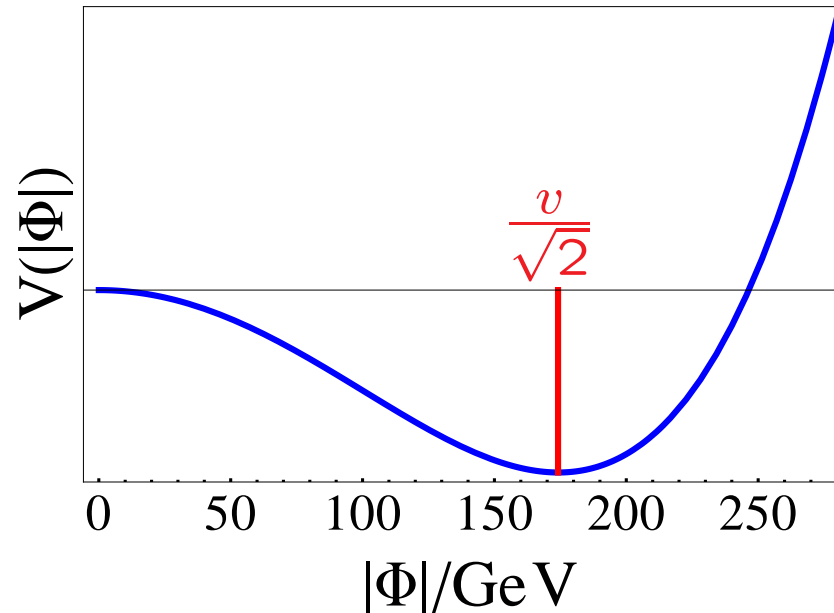
## Classical Higgs potential:

$$V(\Phi) = m^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

$$|\Phi(x)| = \frac{1}{\sqrt{2}} (v + H(x))$$

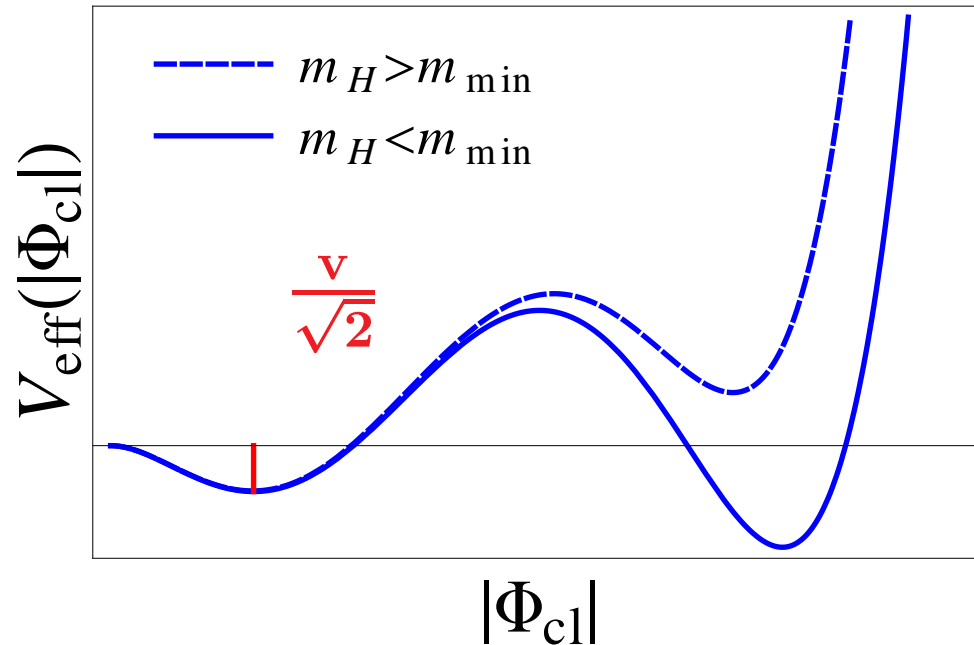
$$\Phi_{\text{cl}}(x) \equiv \langle 0 | \Phi(x) | 0 \rangle = \frac{v}{\sqrt{2}} \neq 0$$

$$v \approx 246 \text{ GeV}, M_H^2 = -2m^2 = 2\lambda v^2$$



# The effective Potential

- Radiative corrections  $\Rightarrow$   $V_{\text{eff}}(\lambda(\Lambda), g_i(\Lambda), y_t(\Lambda), \dots) [\Phi(\Lambda)]$  [Coleman, Weinberg]  
 ( $\Lambda$ : scale up to which the SM is valid)



- $\Phi_{\text{cl}} \sim \Lambda \gg v$ :  $V_{\text{eff}}^{\text{RG}}[\Phi] = \lambda(\Lambda)\Phi^4(\Lambda) + \mathcal{O}(\lambda^2(\Lambda), g_i^2(\Lambda))$  [Altarelli, Isidori; Ford, Jack, Jones]
- Stability of SM vacuum  $\Leftrightarrow \lambda(\Lambda) > 0$  [Cabibbo; Sher; Lindner; Ford]

# Evolution of couplings $X \in \{\lambda, g_1, g_2, g_s, y_t, \dots\}$

$\beta$ -functions:  $\mu^2 \frac{d}{d\mu^2} X(\mu^2) = \beta_X[\lambda(\mu^2), y_t(\mu^2), g_i(\mu^2), \dots]$

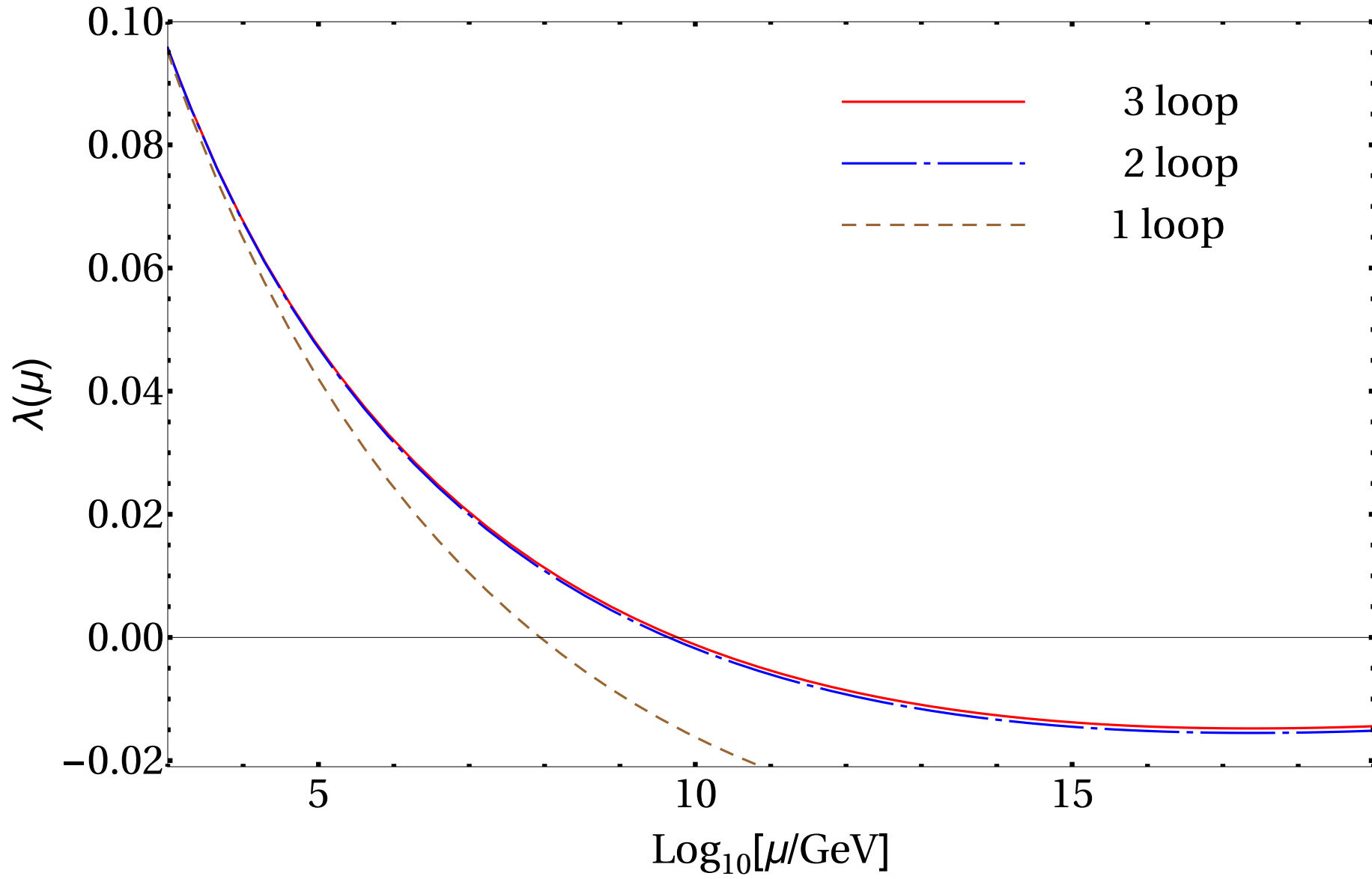
$\Rightarrow$  Coupled system of differential equations with initial conditions:

$$\begin{aligned} \mu^2 \frac{d}{d\mu^2} \lambda(\mu^2) &= \beta_\lambda[\lambda(\mu^2), y_t(\mu^2), g_i(\mu^2)], & \lambda(\mu_0^2) &= \lambda_0, \\ \mu^2 \frac{d}{d\mu^2} y_t(\mu^2) &= \beta_{y_t}[\lambda(\mu^2), y_t(\mu^2), g_i(\mu^2)], & y_t(\mu_0^2) &= y_{t0}, \\ \mu^2 \frac{d}{d\mu^2} g_s(\mu^2) &= \beta_{g_s}[\lambda(\mu^2), y_t(\mu^2), g_i(\mu^2)], & g_s(\mu_0^2) &= g_{s0}, \\ \mu^2 \frac{d}{d\mu^2} g_2(\mu^2) &= \beta_{g_2}[\lambda(\mu^2), y_t(\mu^2), g_i(\mu^2)], & g_2(\mu_0^2) &= g_{20}, \\ \mu^2 \frac{d}{d\mu^2} g_1(\mu^2) &= \beta_{g_1}[\lambda(\mu^2), y_t(\mu^2), g_i(\mu^2)], & g_1(\mu_0^2) &= g_{10} \end{aligned}$$

Calculated in  $\overline{\text{MS}}$ -scheme,  
power series in couplings

Experimental data matched to  
 $\overline{\text{MS}}$ -scheme

# Evolution of $\lambda(\mu)$



## II. Standard Model $\beta$ -functions up to 4 loops

### 2 loop

[M. Fischler, C. Hill (1981); D. Jones (1982); M. Machacek, M. Vaughn (1983,1984,1985); I. Jack, H. Osborn (1984,1985)] [M. Fischler, J. Oliensis (1982); M. Machacek, M. Vaughn (1984); C. Ford, I. Jack, D. Jones (1992); M. Luo, Y. Xiao (2003)]

### 3 loop

- for gauge couplings  $g_1, g_2, g_s$  [L. Mihaila, J. Salomon, M. Steinhauser (2012); A. Bednyakov, A. Pikelner, Velizhanin (2012)]
- for Yukawa couplings  $y_t, y_b, y_\tau$ , etc. [K. Chetyrkin, M.Z. (2012); A. Bednyakov, A. Pikelner, Velizhanin (2013)]
- for the Higgs self-coupling  $\lambda$  (and the mass parameter  $m^2$ ) [K. Chetyrkin, M.Z. (2012 and 2013); A. Bednyakov, A. Pikelner, Velizhanin (2013)]

### 4 loop

- $\beta_{g_s}(g_s)$  [T. van Ritbergen, J. Vermaseren, S. Larin (1997); M. Czakon (2005)]
- $\beta_{g_s}(g_s, y_t, \lambda)$  [A. Bednyakov, A. Pikelner (2015); M.Z. (2015)]
- $\beta_\lambda \propto y_t^4 g_s^6$  [S. Martin (2016); K. Chetyrkin, M.Z. (2016)]
- $\beta_{y_t} \propto y_t g_s^8$  and  $\beta_{m^2} \propto y_t^2 g_s^6$  [K. Chetyrkin, M.Z. (2016)]

# Calculation of $\beta$ -functions, e.g. $\beta_\lambda(\lambda, g_i)$

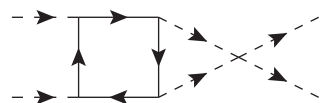
$$\text{Diagram} = \underbrace{\text{Diagram}}_{\propto \frac{y_t^4}{(16\pi^2)}} + \underbrace{\text{Diagram} + \text{Diagram}}_{\propto \frac{\lambda^2}{(16\pi^2)}} + \dots + \text{Diagram} = \text{finite}$$

$$\left[ \text{Diagram} = \text{finite} \right] \Rightarrow \text{Field strength renormalization constant } Z_2^{(2H)}$$

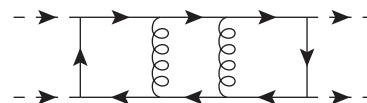
$$\lambda_B = \mu^{2\epsilon} (\lambda + \delta Z_\lambda); \quad \delta Z_\lambda = (\lambda - \delta Z_1^{(4H)}) \left( Z_2^{(2H)} \right)^{-2} = \sum_{n=1}^{\infty} \frac{a_n(\lambda, g_i)}{\epsilon^n}$$

$$\text{use } \mu^2 \frac{d}{d\mu^2} \lambda_B \equiv 0 \Rightarrow \beta_\lambda = \left[ \lambda \frac{\partial}{\partial \lambda} + \frac{1}{2} \sum_i g_i \frac{\partial}{\partial g_i} - 1 \right] a_1$$

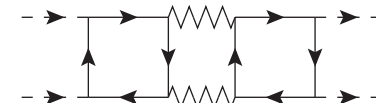
higher orders:



$$\propto \frac{y_t^4 \lambda}{(16\pi^2)^3}$$



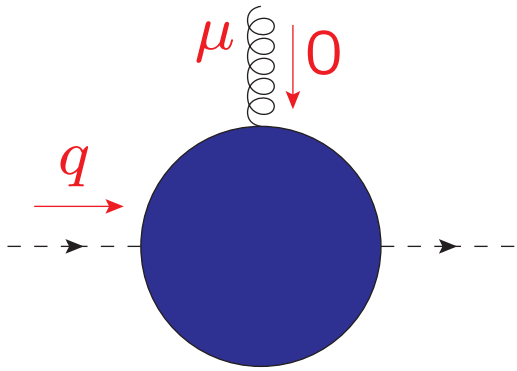
$$\propto \frac{y_t^4 g_s^4}{(16\pi^2)^3}$$



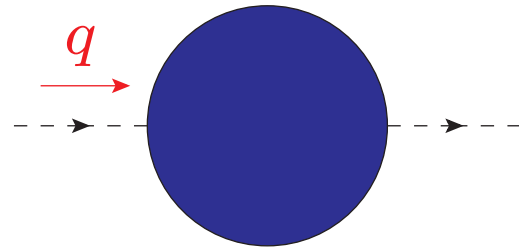
$$\propto \frac{y_t^4 g_2^2}{(16\pi^2)^3}$$



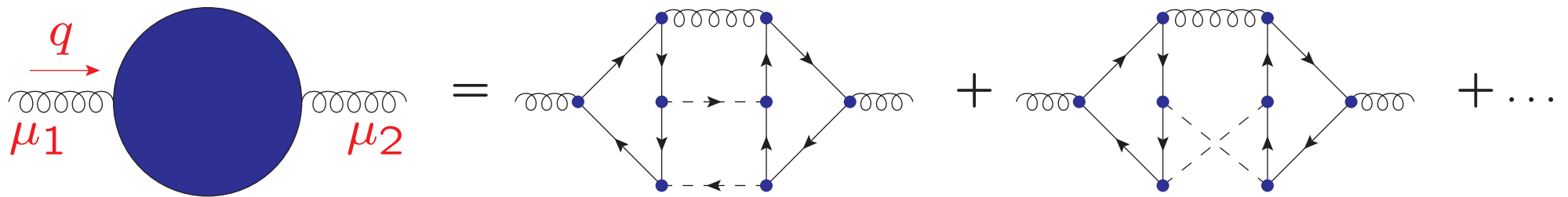
# $\beta_{g_s}$ at 4 loop from the ghost-gluon-vertex



projector:  $\frac{q^\mu}{q^2}$



already scalar  $\propto q^2$



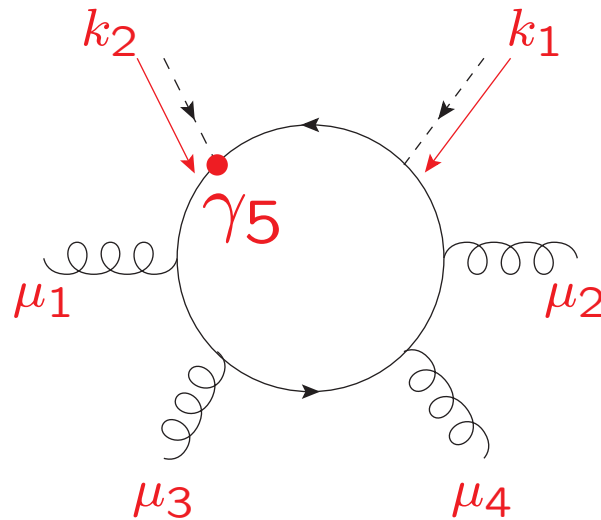
projector:  $\frac{1}{D-1} g^{\mu_1 \mu_2} \cdot \text{Ptr} + q^{\mu_1} q^{\mu_2} \cdot \text{Plong}$

# Treatment of $\gamma_5$

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \frac{i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$$

$$\text{in } D = 4: \{\gamma_5, \gamma^\mu\} = 0 \text{ and } \gamma_5^2 = 1$$

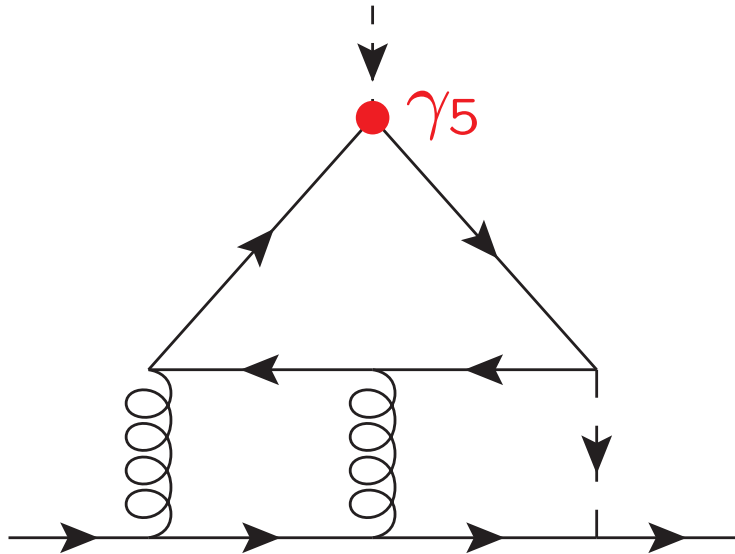
fermion loop:



$$\text{Tr}(\dots) \propto \# \epsilon_{\mu_1\mu_2\mu_3\mu_4} + \# \epsilon_{\mu_1\mu_2\alpha\beta} k_1^\alpha k_2^\beta + \dots$$

$\Rightarrow$  At least 4 free Lorentz structures needed, else diagram=0

# Treatment of $\gamma_5$ in the $Z_{yt}$ calculation



$$\propto P_L = \frac{1}{2}(1 - \gamma_5)$$

use projector  $\propto \gamma_5$  on external fermion line, apply

$$\gamma_5 = \frac{i}{4!} \epsilon_{\mu\nu\rho\sigma} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \text{ with } \epsilon_{0123} = 1$$

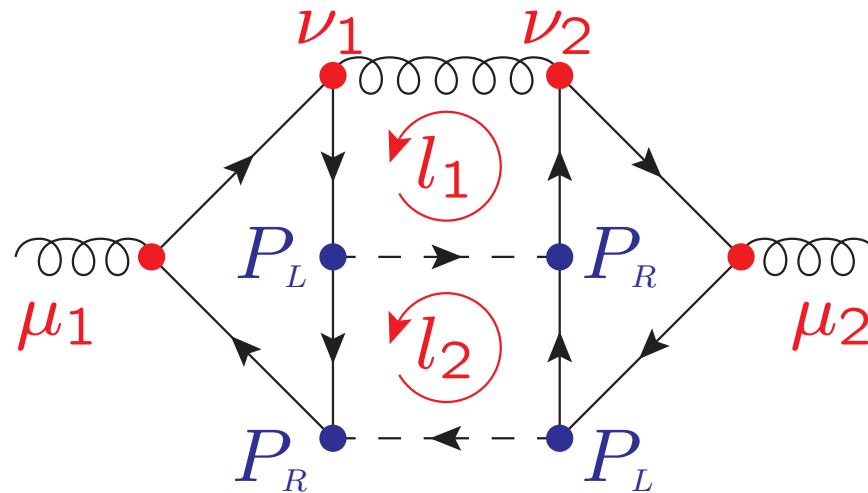
and use

$$\epsilon^{\mu_1\mu_2\mu_3\mu_4} \epsilon_{\nu_1\nu_2\nu_3\nu_4} = -g^{\mu_1\nu_1} g^{\mu_2\nu_2} g^{\mu_3\nu_3} g^{\mu_4\nu_4}$$

(error of  $O(\epsilon)$ )

$\Rightarrow$  Pole part OK if Feynman integrals have only  $\frac{1}{\epsilon}$  poles. ( $\checkmark$ )

# Treatment of $\gamma_5$ in the gluon propagator



- Use  $\varepsilon^{\mu_1\mu_2\mu_3\mu_4}\varepsilon_{\nu_1\nu_2\nu_3\nu_4} = -g_{\nu_1}^{[\mu_1}g_{\nu_2}^{\mu_2}g_{\nu_3}^{\mu_3}g_{\nu_4}^{\mu_4]} (1 + \varepsilon \cdot \text{LABEL})$
- Anticommuting  $\gamma_5$  to different points in different diagrams changes result in this case! UV divergent part always transversal.
- Move  $\gamma_5$  to external vertices  $\rightarrow \frac{1}{\varepsilon}g_s^4y_t^4T_F^2 \left(\frac{4}{3} + 8\zeta_3\right)$
- Leave  $\gamma_5$  at their original position  $\rightarrow 3 \cdot \frac{1}{\varepsilon}g_s^4y_t^4T_F^2 \left(\frac{4}{3} + 8\zeta_3\right)$   
Only here also finite part of self-energy transversal [Bednyakov, Pikelner]

# Automation

- Generation of diagrams → QGRAF [Nogueira]
- $SU(2) \times U_Y(1)$  group factors → Form code [M.Z.]
- $SU_C(3)$  group factors → COLOR [Van Ritbergen, Schellekens, Vermaseren]
- find topologies → GEFICOM [Chetyrkin, M.Z.] and  
Q2E, EXP [Seidesticker, Harlander, Steinhauser]
- Feynman rules, projectors, counterterms, fermion traces, expansion in external momenta → FORM [Vermaseren] code [Chetyrkin, M.Z.]
- massive tadpole integrals → up to 3 loop: MATAD [Steinhauser];  
Reduction at 4 loop: FIRE5 (C++ version) [Smirnov] (based on IBP)  
⇒ use 19 Master integrals [Czakon et al]

# Results

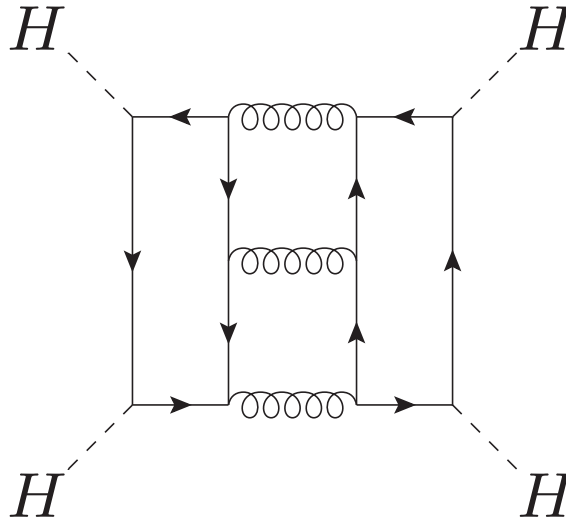
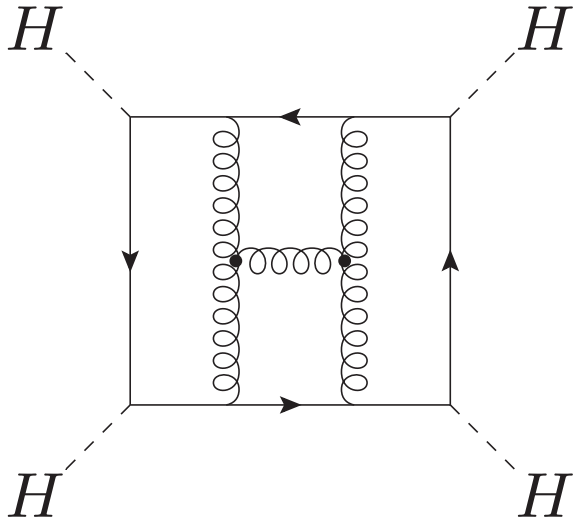
$$\mu^2 \frac{d}{d\mu^2} g_s(\mu) = \beta_{g_s} = \sum_{n=1}^{\infty} \frac{1}{(16\pi^2)^n} \beta_{g_s}^{(n)} \quad (\text{in the } \overline{\text{MS}}\text{-scheme})$$

$$\begin{aligned} \frac{\beta_{g_s}^{(4)}}{g_s} = & + g_s^8 \left( -\frac{149753}{12} + \frac{1078361}{324} n_f - \frac{50065}{324} n_f^2 - \frac{1093}{1458} n_f^3 \right. \\ & \left. - 1782 \zeta_3 + \frac{3254}{27} \zeta_3 n_f - \frac{3236}{81} \zeta_3 n_f^2 \right) \\ & + g_s^6 y_t^2 \left( -\frac{9959}{18} + \frac{1625}{54} n_f + 136 \zeta_3 \right) \\ & + g_s^4 y_t^4 \left( \frac{423}{2} + 2 - 60 \zeta_3 + 12 \zeta_3 \right) \quad \leftarrow \text{non-naive part} \\ & + g_s^2 y_t^6 \left( -\frac{423}{8} - 3 \zeta_3 \right) - 15 g_s^2 y_t^4 \lambda + 18 g_s^2 y_t^2 \lambda^2 \end{aligned}$$

$$\begin{aligned} \frac{\beta_{g_s}^{(4)}}{\beta_{g_s}^{(1)} (16\pi^2)^3} = & \underbrace{2.26 \times 10^{-4}}_{g_s^8} + \underbrace{2.47 \times 10^{-5}}_{g_s^6 y_t^2} - \underbrace{1.06 \times 10^{-5}}_{g_s^4 y_t^4 (\text{naive})} + \underbrace{12.51 \times 10^{-7}}_{g_s^4 y_t^4 (\text{non-naive})} \\ & \underbrace{+ 2.77 \times 10^{-6}}_{g_s^2 y_t^6} + \underbrace{+ 1.06 \times 10^{-7}}_{g_s^2 y_t^4 \lambda} - \underbrace{1.82 \times 10^{-8}}_{g_s^2 y_t^2 \lambda^2} \end{aligned}$$

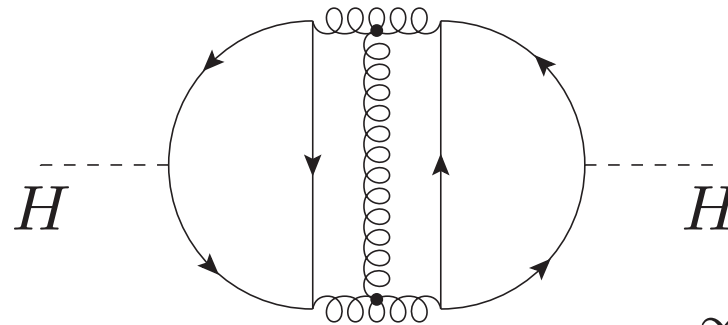
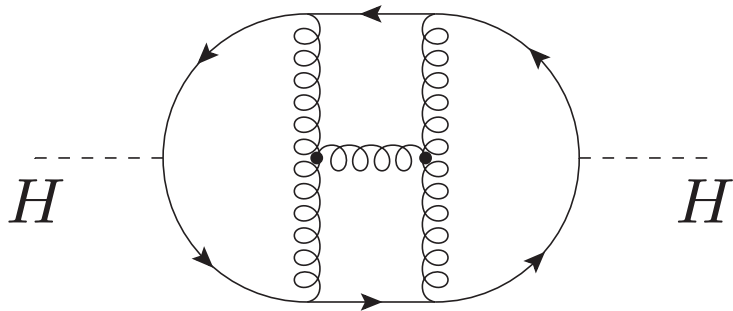
# Dominant contribution to $\beta_\lambda$ at four loops

## Four-Higgs-vertex:



$$\propto \frac{y_t^4 g_s^6}{(16\pi^2)^4}$$

## Higgs propagator:



$$\propto \frac{y_t^2 g_s^6}{(16\pi^2)^4}$$

# Results

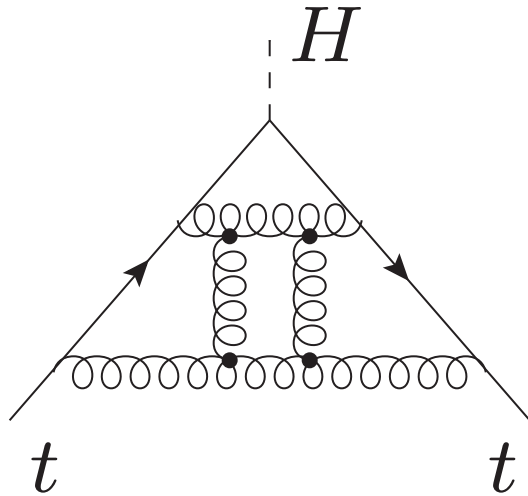
$$\mu^2 \frac{d}{d\mu^2} \lambda(\mu) = \beta_\lambda = \sum_{n=1}^{\infty} \frac{1}{(16\pi^2)^n} \beta_\lambda^{(n)} \quad (\text{in the } \overline{\text{MS}}\text{-scheme})$$

$$\begin{aligned} \beta_\lambda^{(4)} = & y_t^4 g_s^6 d_R \left\{ C_F^3 \left( -\frac{2942}{3} + 160\zeta_5 + 288\zeta_4 + 48\zeta_3 \right) \right. \\ & + T_F C_F^2 \left( -64 + n_f \left( +\frac{562}{3} - 160\zeta_4 + \frac{32}{3}\zeta_3 \right) \right) \\ & + C_A C_F^2 \left( \frac{3584}{3} + 720\zeta_5 + 32\zeta_4 - \frac{3304}{3}\zeta_3 \right) \\ & + C_A T_F C_F \left( \frac{5888}{9} - 160\zeta_5 + 352\zeta_3 + n_f \left( -\frac{2644}{243} + 128\zeta_4 + 16\zeta_3 \right) \right) \\ & + C_A^2 C_F \left( -\frac{121547}{243} - 520\zeta_5 - 88\zeta_4 + \frac{1880}{3}\zeta_3 \right) \\ & \left. + T_F^2 C_F \left( -\frac{256}{9}n_f + n_f^2 \left( -\frac{128}{3}\zeta_3 + \frac{10912}{243} \right) \right) \right\} \\ & + \mathcal{O}(y_t^6) + \mathcal{O}(\lambda) + \mathcal{O}(g_2) + \mathcal{O}(g_1) \end{aligned}$$



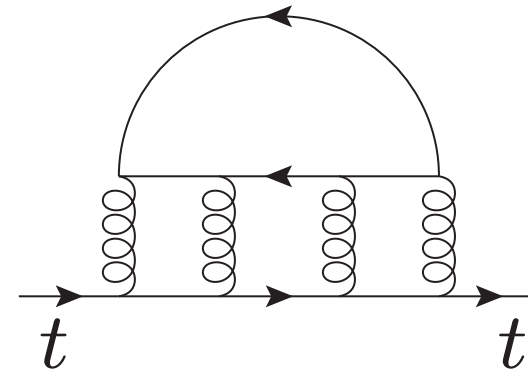
# Dominant contribution to $\beta_{y_t}$ at four loops:

Higgs- $t\bar{t}$ -vertex:



$$\propto \frac{y_t g_s^8}{(16\pi^2)^4}$$

Top-propagator:



$$\propto \frac{g_s^8}{(16\pi^2)^4}$$

# Results

$$\mu^2 \frac{d}{d\mu^2} y_t(\mu) = \beta_{y_t} = \sum_{n=1}^{\infty} \frac{1}{(16\pi^2)^n} \beta_{y_t}^{(n)} \quad (\text{in the } \overline{\text{MS}}\text{-scheme})$$

$$\begin{aligned} \beta_{y_t}^{(4)} = & y_t g_s^8 \left\{ \frac{d_F^{abcd} d_A^{abcd}}{d_R} (32 - 240\zeta_3) + n_f \frac{d_F^{abcd} d_F^{abcd}}{d_R} (-64 + 480\zeta_3) \right. \\ & + C_F^4 \left( \frac{1261}{8} + 336\zeta_3 \right) - C_A C_F^3 \left( \frac{15349}{12} + 316\zeta_3 \right) \\ & + C_A^2 C_F^2 \left( \frac{34045}{36} - 440\zeta_5 + 152\zeta_3 \right) + C_A^3 C_F \left( -\frac{70055}{72} + 440\zeta_5 - \frac{1418}{9}\zeta_3 \right) \\ & + n_f T_F C_F^3 \left( \frac{280}{3} + 480\zeta_5 - 552\zeta_3 \right) + n_f C_A T_F C_F^2 \left( \frac{8819}{27} - 80\zeta_5 + 264\zeta_4 - 368\zeta_3 \right) \\ & + n_f C_A^2 T_F C_F \left( \frac{65459}{162} - 400\zeta_5 - 264\zeta_4 + \frac{2684}{3}\zeta_3 \right) \\ & + n_f^2 T_F^2 C_F^2 \left( -\frac{304}{27} - 96\zeta_4 + 160\zeta_3 \right) + n_f^2 C_A T_F^2 C_F \left( -\frac{1342}{81} + 96\zeta_4 - 160\zeta_3 \right) \\ & \left. + n_f^3 T_F^3 C_F \left( \frac{664}{81} - \frac{128}{9}\zeta_3 \right) \right\} + \mathcal{O}(y_t^3) + \mathcal{O}(\lambda) + \mathcal{O}(g_2) + \mathcal{O}(g_1). \end{aligned}$$

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with  $d_F^{abcd} = \frac{1}{6} \text{Tr} (T^a T^b T^c T^d + T^a T^b T^d T^c + T^a T^c T^b T^d + T^a T^c T^d T^b + T^a T^d T^b T^c + T^a T^d T^c T^b)$

# Beyond the SM: $\beta_{\alpha_s}$ with different fermion representations

$$\begin{aligned} \mathcal{L}_{QCD} = & -\frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu} - \frac{1}{2(1-\xi)} (\partial_\mu A^{a\mu})^2 + \partial_\mu \bar{c}^a \partial^\mu c^a + g_s f^{abc} \partial_\mu \bar{c}^a A^{b\mu} c^c \\ & + \sum_{r=1}^{N_{\text{rep}}} \sum_{q_r} \left\{ \frac{i}{2} \bar{q}_r \overleftrightarrow{\not{D}} q_r + g_s \bar{q}_r A^a T_r^a q_r \right\} \end{aligned}$$

## Gauge group factors:

$r = 1, \dots, N_{\text{rep}}$  fermion representations of dimension  $d_{F,i}$ ,

Adjoint representation with dimension  $N_A$ ,

Casimirs:  $C_{F,i}$  for fermion representation,  $C_A$  for adjoint

$$\begin{aligned} T_{F,i} \delta^{ab} &= \text{Tr} (T^{a,i} T^{b,i}) \\ d_i^{a_1 a_2 \dots a_n} &= \frac{1}{n!} \sum_{\text{perm } \pi} \text{Tr} \{ T^{a_{\pi(1)},i} T^{a_{\pi(2)},i} \dots T^{a_{\pi(n)},i} \}, \end{aligned}$$

## Application: QCD + gluinos (Majorana fermions)

$$\begin{aligned} n_{f,1} &= n_f, & n_{f,2} &= \frac{n_{\tilde{g}}}{2}, \\ T_{F,1} &= T_F, & T_{F,2} &= C_A, \\ C_{F,1} &= C_F, & C_{F,2} &= C_A \end{aligned}$$

# $\beta_{\alpha_s}$ with different fermion representations

$$\beta_{\alpha_s}^{(1)} = -\frac{11}{3}C_A + \sum_i \frac{4}{3}n_{f,i}T_{F,i} \quad (1)$$

$$\begin{aligned} & \vdots \\ \beta_{\alpha_s}^{(4)} &= -\left(\frac{150653}{486} - \frac{44}{9}\zeta_3\right)C_A^4 + \left(\frac{80}{9} - \frac{704}{3}\zeta_3\right)\frac{d_A^{abcd}d_A^{abcd}}{N_A} \\ &+ \sum_i n_{f,i}T_{F,i} \left[ -46C_{F,i}^3 + \left(\frac{4204}{27} - \frac{352}{9}\zeta_3\right)C_A C_{F,i}^2 - \left(\frac{7073}{243} - \frac{656}{9}\zeta_3\right)C_A^2 C_{F,i} \right. \\ &\quad \left. + \left(\frac{39143}{81} - \frac{136}{3}\zeta_3\right)C_A^3 \right] - \sum_i n_{f,i} \left(\frac{512}{9} - \frac{1664}{3}\zeta_3\right)\frac{d_{F,i}^{abcd}d_A^{abcd}}{N_A} \\ &+ \sum_{i,j} n_{f,i}n_{f,j}T_{F,i}T_{F,j} \left[ -\left(\frac{184}{3} - 64\zeta_3\right)C_{F,i}C_{F,j} + \left(\frac{304}{27} + \frac{128}{9}\zeta_3\right)C_{F,i}^2 \right. \\ &\quad \left. - \left(\frac{17152}{243}C_A C_{F,i} + \frac{448}{9}\zeta_3\right)C_A C_{F,i} - \left(\frac{7930}{81} + \frac{224}{9}\zeta_3\right)C_A^2 \right] \\ &+ \sum_{i,j} n_{f,i}n_{f,j} \left(\frac{704}{9} - \frac{512}{3}\zeta_3\right)\frac{d_{F,i}^{abcd}d_{F,j}^{abcd}}{N_A} \\ &- \sum_{i,j,k} n_{f,i}n_{f,j}n_{f,k}T_{F,i}T_{F,j}T_{F,k} \left[ \frac{1232}{243}C_{F,i} + \frac{424}{243}C_A \right] \end{aligned} \quad (2)$$

# IV. Current status of vacuum stability

## Starting values for SM couplings

[Sirlin, Zucchini; Hempfling, Kniehl; Jegerlehner et al; Bezrukov et al; Buttazzo et al; Kniehl, Veretin]

### Example: The Higgs propagator

1 PI self energy:

$$\text{---} \circ \text{---} + \text{---} \circ \text{---} + \dots =: \text{---} \bigcirc \text{---} =: \Sigma(p^2, m_H, g_i)$$

full propagator:

$$\text{---} + \text{---} \bigcirc \text{---} + \text{---} \bigcirc \bigcirc \text{---} + \dots = \frac{i}{p^2 - m_H^2 - \Sigma(p^2, m_H, g_i)}$$

Physical mass  $M_H \Leftrightarrow$  pole of full propagator

$$\text{For } p^2 = M_H^2 \Rightarrow m_H^2 - \Sigma(p^2, m_H, g_i) = M_H^2 \text{ with } m_H = \sqrt{2\lambda} v$$

Same for  $M_t, M_W, M_Z, G_F \Rightarrow$  Solve for  $\lambda, v, y_t$ , etc.

# Starting values for SM couplings

Experimental input

$$M_t^{\text{MC}} \approx 173.34 \pm 0.76 \text{ GeV} \Rightarrow M_t = 173.39_{-0.98}^{+1.12} \text{ GeV}$$

[Moch et al] via MSR mass [Hoang et al]

$$M_H \approx 125.09 \pm 0.24 \text{ GeV}$$

$$\alpha_s \approx 0.1185 \pm 0.0006$$

$\overline{\text{MS}}$ -couplings:

$$g_s(M_t) = 1.1652 \pm 0.0035(\text{exp}),$$

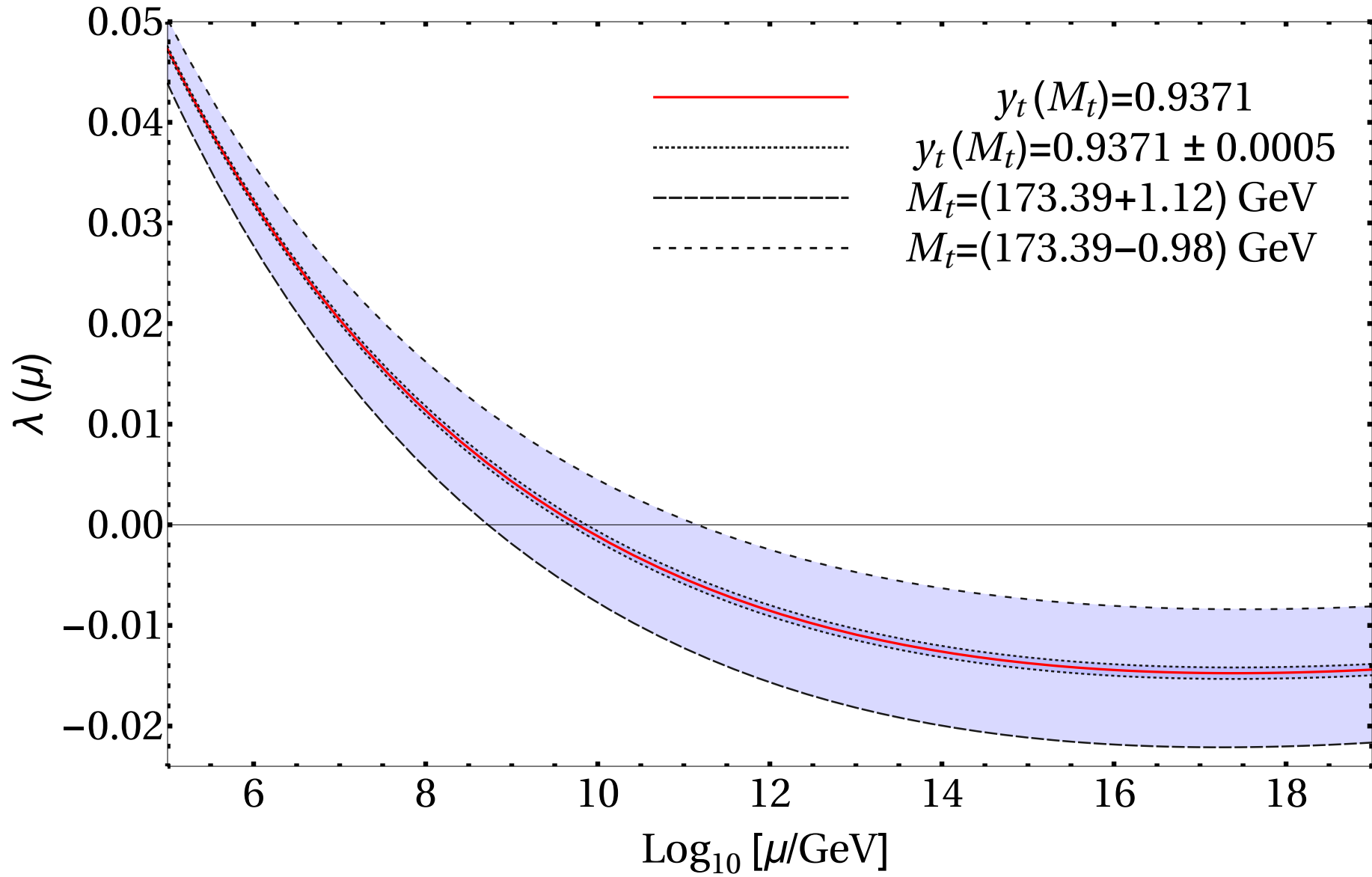
$$y_t(M_t) = 0.9374_{-0.0062}^{+0.0063}(\text{exp}) \pm 0.0005 \text{ (2 loop matching),}$$

$$\lambda(M_t) = 0.1259 \pm 0.0005(\text{exp}) \pm 0.0003 \text{ (2 loop matching),}$$

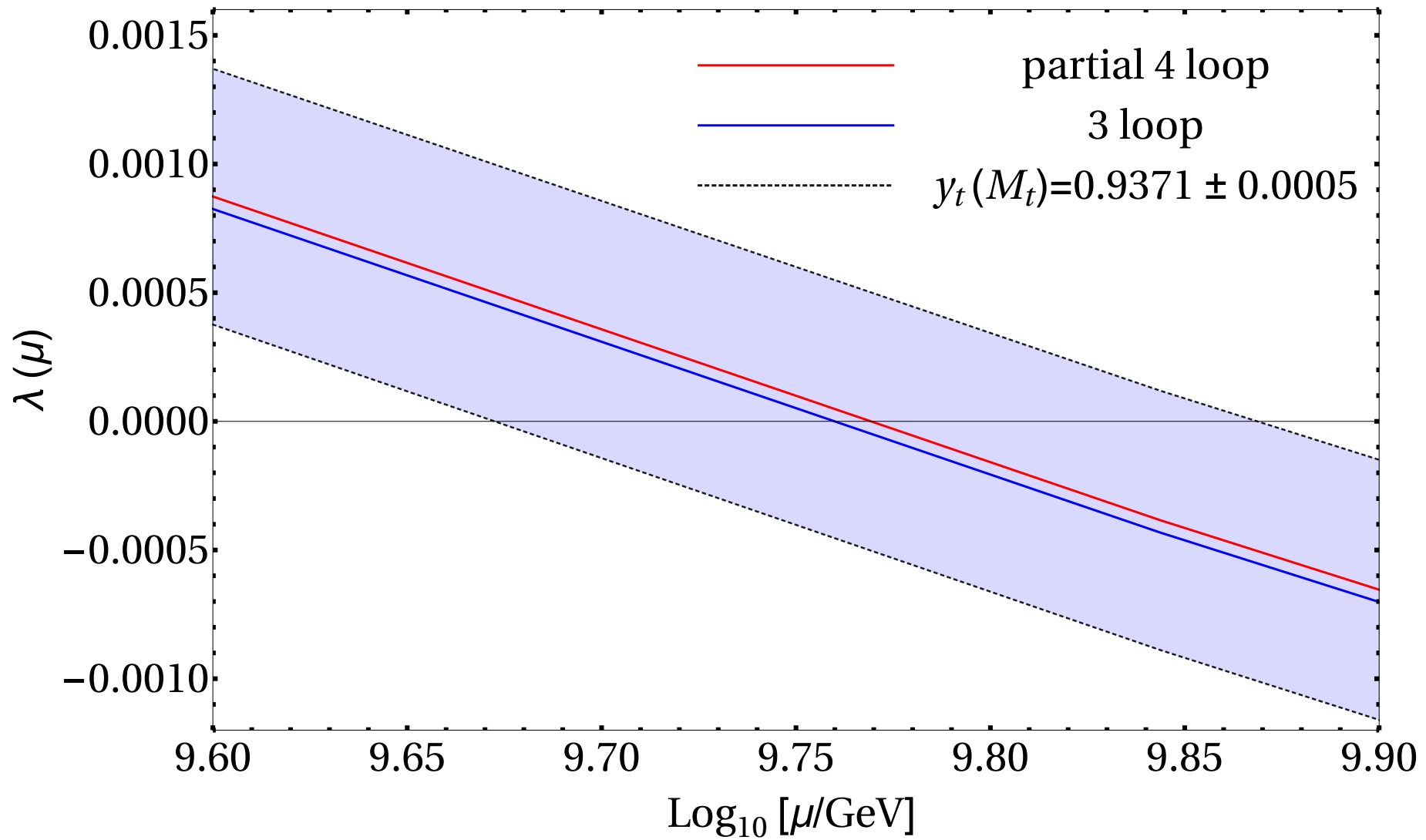
$$g_2(M_t) = 0.6483,$$

$$g_1(M_t) = 0.3587$$

# Evolution of $\lambda(\mu)$



# Evolution of $\lambda(\mu)$





# Summary

- Stability of SM vacuum  $\leftrightarrow$   $\lambda > 0$
- $\beta$ -functions in full SM at 3 loop and now partial 4 loop
- Limiting problem for full 4 loop  $\beta$ -functions and 3 loop matching in the SM:  $\gamma_5$
- SM vacuum looks not stable (metastable),  
 $M_t$  largest source of uncertainty

BACKUP

# Evolution of $\lambda(\mu)$

